



# **New International Formulation for the Thermal Conductivity of H2O**

[M. L. Huber,](http://scitation.aip.org/search?value1=M.+L.+Huber&option1=author) [R. A. Perkins](http://scitation.aip.org/search?value1=R.+A.+Perkins&option1=author), [D. G. Friend](http://scitation.aip.org/search?value1=D.+G.+Friend&option1=author), [J. V. Sengers,](http://scitation.aip.org/search?value1=J.+V.+Sengers&option1=author) [M. J. Assael,](http://scitation.aip.org/search?value1=M.+J.+Assael&option1=author) [I. N. Metaxa](http://scitation.aip.org/search?value1=I.+N.+Metaxa&option1=author), [K. Miyagawa,](http://scitation.aip.org/search?value1=K.+Miyagawa&option1=author) [R. Hellmann](http://scitation.aip.org/search?value1=R.+Hellmann&option1=author) , and [E. Vogel](http://scitation.aip.org/search?value1=E.+Vogel&option1=author)

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# New International Formulation for the Thermal Conductivity of H<sub>2</sub>O

M. L. Huber<sup>a)</sup> R. A. Perkins, and D. G. Friend

Thermophysical Properties Division, NIST, Boulder, Colorado 80305, USA

#### J. V. Sengers

Institute for Physical Science and Technology, University of Maryland, College Park, Maryland 20742, USA and Thermophysical Properties Division, NIST, Boulder, Colorado 80305, USA

## M. J. Assael and I. N. Metaxa

Faculty of Chemical Engineering, Aristotle University, GR-54124 Thessaloniki, Greece

#### K. Miyagawa

Arakawa Ku, 4-12-11-628, Tokyo 1160011, Japan

### R. Hellmann and E. Vogel

Institut für Chemie, Universität Rostock, D-18059 Rostock, Germany

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The International Association for the Properties of Water and Steam (IAPWS) encouraged an extensive research effort to update the IAPS Formulation 1985 for the Thermal Conductivity of Ordinary Water Substance, leading to the adoption of a Release on the IAPWS Formulation 2011 for the Thermal Conductivity of Ordinary Water Substance. This paper describes the development and evaluation of the 2011 formulation, which provides a correlating equation for the thermal conductivity of water for fluid states from the melting temperature up to 1173 K and 1000 MPa with uncertainties from less than 1% to 6%, depending on the state point.  $\odot$  2012 by the U.S. Secretary of Commerce on behalf of the United States. All rights reserved. [\[http://dx.doi.org/10.1063/1.4738955\]](http://dx.doi.org/10.1063/1.4738955)

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a)Electronic mail: [marcia.huber@nist.gov.](mailto:marcia.huber@nist.gov)

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## 1. Introduction

An accurate characterization of the thermophysical properties of water and steam  $(H<sub>2</sub>O)$  is very important for both science and technology. For this purpose, the International Association for the Properties of Water and Steam (IAPWS) tries to develop and standardize state-of-the art formulations for the thermophysical properties of  $H<sub>2</sub>O$ . The most recent IAPWS formulations for the thermodynamic properties of H2O are the IAPWS Formulation 1995 for the Thermodynamic Properties of Ordinary Water Substance for General and Scientific  $Use<sup>1,2</sup>$  $Use<sup>1,2</sup>$  $Use<sup>1,2</sup>$  (referred to as IAPWS-95) and the IAPWS Industrial Formulation 1997 for the Thermodynamic Properties of Water and Steam<sup>3,4</sup> (referred to as IAPWS-IF97). After these formulations for the thermodynamic properties had been adopted, IAPWS coordinated an international research project in collaboration with the International Association for Transport Properties (IATP) to also develop updated formulations for the transport properties of  $H_2O$ . These efforts led to the adoption of the IAPWS Formulation 2008 for the Viscosity of Ordinary Water Substance<sup>[5](#page-21-0)</sup> and of the IAPWS Formulation 2011 for the Thermal Conductivity of Ordinary Water Sub-stance.<sup>[6](#page-21-0)</sup> The new formulation for the viscosity of  $H_2O$  was described in a previous paper in this journal.<sup>[7](#page-21-0)</sup> The present paper is concerned with the updated recommendations for calculating the thermal conductivity of  $H_2O$ .

A first set of skeleton tables for the thermal conductivity of H2O, covering pressures up to 50 MPa and temperatures up to 700 °C, together with some interpolating equations excluding the critical region, was issued in 1964 upon the instruction of the 6th International Conference on the Properties of Steam.<sup>8,9</sup> It was not clear at that time how to represent the thermal conductivity of  $H_2O$  in the critical region. The presence of a pronounced critical enhancement of the thermal conductivity

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in fluids, such as carbon dioxide, had just been discovered.<sup>[10,11](#page-21-0)</sup> However, the research group of Amirkhanov et al. had not been able to confirm the presence of a pronounced critical thermal-conductivity enhancement for carbon dioxide<sup>12</sup> or for steam.<sup>[13](#page-21-0)</sup> Hence, the possibility of such an enhancement in steam was discarded, and the thermal conductivity of  $H_2O$  in the critical region was estimated from smooth graphical interpolations along isobars from low temperatures to high temperatures.

In the 1970s, it was demonstrated experimentally that the thermal conductivity of  $H<sub>2</sub>O$  also exhibits a pronounced enhancement in the critical region.<sup>[14,15](#page-21-0)</sup> The 8th International Conference on the Properties of Steam, held in 1974, established a special committee for the purpose of completing new formulations for the transport properties of  $H_2O$ . This effort led to the adoption of a Release on the Dynamic Viscosity of Water Substance in 1975, as was discussed in a previous recent publication,<sup>[7](#page-21-0)</sup> and a Release on the Thermal Conductivity of Water Substance in 1977.<sup>[16–19](#page-21-0)</sup> The 1977 Release contained a table of critically evaluated thermal-conductivity data covering pressures up to 100 MPa and temperatures up to 800 °C, a table of critically evaluated thermal-conductivity data of the saturated vapor and liquid as a function of temperature, a recommended interpolating equation for industrial use in which the thermal conductivity remained finite at the critical point, and a recommended interpolating equation for scientific use in which the thermal conductivity diverges at the critical point as predicted by theory. Both interpolating equations yield the thermal conductivity of  $H_2O$  as a function of density and temperature. The interpolating equation for scientific use, developed by Sengers et  $a l$ , <sup>[19](#page-21-0)</sup> was to be used in conjunction with the 1968 IFC Formulation for Scientific and General  $Use^{20,21}$  $Use^{20,21}$  $Use^{20,21}$  for the thermodynamic properties, while the interpolating equation for industrial use, developed by Yata and Minamiyama, $22$  was to be used in conjunction with the 1967 IFC Formulation for Industrial Use<sup>[23](#page-21-0)</sup> for the thermodynamic properties. The 1968 IFC Formulation was replaced by a Provisional IAPS Formulation 1982 for the Thermodynamic Properties of Ordinary Water Substance for Scientific and General Use based on an equation of state developed by Haar *et al.*,<sup>[24](#page-21-0)</sup> which was subsequently recast in dimensionless form as the IAPWS 1984 Formulation for the Thermodynamic Properties of Ordinary Water Substance.<sup>[25](#page-21-0)</sup> It was certified by the International Association for the Properties of Steam (IAPS) that the recommended interpolating equation for the thermal conductivity for scientific use could be used in conjunction with the IAPWS 1984 Formulation for the Thermodynamic Properties without any loss of accuracy.[19,26](#page-21-0) This formulation was issued as the Release on the IAPS Formulation 1985 for the Thermal Conductivity of Ordinary Water Substance.<sup>[27](#page-21-0)</sup>

As mentioned earlier, the current recommended formulations for the thermodynamic properties of  $H_2O$  are the IAPWS-95 and IAPWS-IF97 formulations. Moreover, the International Practical Temperature Scale of 1968 was replaced by the International Temperature Scale of 1990  $(ITS-90).^{28}$  $(ITS-90).^{28}$  $(ITS-90).^{28}$  For these reasons, the IAPS Formulation 1985 for the Thermal Conductivity of Ordinary Water Substance was slightly revised, the last revision having been adopted in 2008 to make the information consistent with both the IAPWS-95 and IAPWS-IF97 formulations for the thermodynamic properties, with ITS-90, and with the recent IAPWS Formulation 2008 for the viscosity.[29](#page-21-0) Nevertheless, we should emphasize that these recommended equations for the thermal conductivity are still the same as the equations adopted in 1977, which are based on the experimental thermal-conductivity information available at that time. $30$ Moreover, these equations do not cover the higher temperatures and pressures covered by the IAPWS-95 formulation for the thermodynamic properties. Hence, as for the viscosity, development of a new formulation for the thermal conductivity of H2O had become desirable, culminating in the adoption of the IAPWS Formulation 2011 for the Thermal Conductivity of Ordinary Water Substance.<sup>[6](#page-21-0)</sup>

## 2. Experimental Data

As part of a joint project between IAPWS and IATP, experimental data on the viscosity and thermal conductivity of water and steam were collected, converted to the ITS-90 temperature scale and a common set of units, and evaluated. Unless the temperature scale was explicitly stated in a publication or additional information was available, the year of publication was used to determine the appropriate temperature scale for the conversion. Thus the assigned temperature associated with each thermal conductivity data point was adjusted. In principle, also the values of the thermal conductivity may be affected by a change in the temperature scale because of its possible influence on the temperature gradient. This issue was investigated by Mares and Kalova, $31$  who concluded that the effect of converting from IPTS-68 to ITS-90 for properties like heat capacity and thermal conductivity never exceeds 0.2% and, therefore, can be neglected when analyzing thermal conductivity data for water. The resultant database for the thermal conductivity of water, containing 5107 points covering the range of temperatures from 256 K to 1191 K with pressures to 785 MPa, is described in detail in Ref. [32](#page-21-0). This database contains data collected through bibliographic efforts under the auspices of IAPWS and documented in unpublished reports through 1988, as well as including later data through the year 2000.

In the work reported here, we used the database described in Ref. [32](#page-21-0) as a starting point. Several works that appeared after the year  $2000$  were added.<sup>[33–36](#page-21-0)</sup> In addition, we included a table of data provided by  $Sirota^{37}$  $Sirota^{37}$  $Sirota^{37}$  to IAPWS that summarizes the results based on a series of papers written from 1973 to 1981 (Refs. [14,38–44](#page-21-0)) and that supersedes the original publications. Furthermore, we calculated thermal conductivities from the thermal-diffusivity data of Abramson.[45](#page-21-0) The Abramson data are at very high pressures (to 4 GPa) that exceed the range of applicability of the IAPWS-95 formulation<sup>[1,2](#page-21-0)</sup> and were not included in the 2000 database. However, we used these data to guide the highpressure extrapolation behavior of the correlation. Finally,

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FIG. 1. Distribution of experimental thermal-conductivity data in the temperature-pressure plane.

there are three sets of thermal-diffusivity data<sup>[46–48](#page-21-0)</sup> that are not in the database, but that were used in this work. Two sets in the supercooled liquid region $46,47$  were used to examine the extrapolated behavior of the formulation in the supercooled region. Thermal conductivity derived from thermaldiffusivity data in the critical region $48$  were also used in the development of the formulation.

The temperature and pressure range of the available data are shown in Fig. 1 along with the range of applicability of the 1985/2008 IAPWS formulation for the thermal conductivity of water.<sup>[29](#page-21-0)</sup> The 2008 formulation is a minor revision of the 1998 release<sup>[49](#page-21-0)</sup> that is itself a minor modification of the formulation adopted in [19](#page-21-0)85.<sup>[1](#page-5-0)9</sup> Table 1 summarizes all data considered in this work, including an estimate of their uncertainty, the temperature and pressure ranges of the data, the experimental method, and the number of individual data points. Densities have been calculated from the IAPWS-95 Formulation for the Thermodynamic Properties of Ordinary Water Substance for General and Scientific Use.<sup>[1,2](#page-21-0)</sup> The resultant dataset used for regression comprises 7162 data points from 103 sources that cover a temperature range from 255.75 K to 1191.1 K and pressures up to 1176.8 MPa.

#### 3. Development of the Correlation

In order to provide consistency with the conventions adopted by IAPWS in its releases on the transport properties of water, we use the following dimensionless variables for temperature T, density  $\rho$ , pressure p, and thermal conductivity λ,

$$
\overline{T} = \frac{T}{T^*}, \quad \overline{\rho} = \frac{\rho}{\rho^*}, \quad \overline{p} = \frac{p}{p^*}, \quad \overline{\lambda} = \frac{\lambda}{\lambda^*}, \quad (1)
$$

where the reference constants are  $T^* = 647.096$  K,  $\rho^* = 322.0$ kg m<sup>-3</sup>,  $p^* = 22.064$  MPa, and  $\lambda^* = 1.00$  mW m<sup>-1</sup> K<sup>-1</sup>. The reference values for temperature, pressure, and density are in agreement with the presently accepted critical parameters recommended by IAPWS.[139](#page-22-0) All temperatures are expressed in terms of the ITS-90 temperature scale.

The formulation for the thermal conductivity of water has the general form $^{26}$  $^{26}$  $^{26}$ 

$$
\overline{\lambda} = \overline{\lambda}_0(\overline{T}) \times \overline{\lambda}_1(\overline{T}, \overline{\rho}) + \overline{\lambda}_2(\overline{T}, \overline{\rho}). \tag{2}
$$

The first factor  $\overline{\lambda}_0$  of the product represents the thermal conductivity in the zero-density limit and is a function of temperature only. The second factor  $\overline{\lambda}_1$  represents the contribution to thermal conductivity due to increasing density, while the third factor  $\overline{\lambda}_2$  represents an enhancement of the thermal conductivity in the critical region. The determination of each of these contributions will be considered in Secs. 3.1–3.3.

#### 3.1. Thermal conductivity in the limit of zero density

In order to develop an expression for the thermal conductivity in the limit of zero density, all points from the database for densities less than 50 kg m<sup> $-3$ </sup> were considered. This resulted in a total of 1008 points from 42 sources. These are denoted in Table [1](#page-5-0) with italics, along with the experimental method used and an estimate of the uncertainty of the mea-surement.<sup>[32](#page-21-0)</sup> As indicated in Table [1](#page-5-0), the measurements were made by several independent groups with different experimental methodologies. We did not select any of the points published by Sirota et al. in journal publications from 1970 to 1981,<sup>[14,38–40,42–44,140–142](#page-21-0)</sup> since they are also incorporated in a report $37$  that summarizes all of the experiments of that time period. This report is considered to have the best values incorporating recommended corrections. We also did not consider the 1968 work of Le Neindre et  $al$ ,<sup>[77](#page-22-0)</sup> since the data reported are dependent upon the IAPS 1964 Skeleton Tables. In addition, we excluded 21 points from Milverton $52$  corresponding to pressures below 13.3 kPa (10 cm Hg) that were considered to be subject to severe temperature jumps.<sup>[19](#page-21-0)</sup> Following the procedure used in the development of the formulation for the viscosity of water, $\frac{7}{1}$  $\frac{7}{1}$  $\frac{7}{1}$  the remaining points were then sorted into temperature "bins" of at least three points that covered temperature ranges up to 8 K and included at least one point with a density less than 10 kg  $\text{m}^{-3}$ ; the average bin size was approximately 4 K. This procedure is based on the concept of Key Comparison Reference Values<sup>[143](#page-23-0)</sup> that incorporates statistical analysis of the data reflecting the uncertainties from different data sources. Some points could not be included in the bins and were removed from consideration. The four data points of Miroshnichenko and Makhrov<sup>[122](#page-22-0)</sup> were treated separately as zero-density points and were not included in bins. The nominal temperature of an isotherm was computed as the average temperature of all points in a bin. The thermal conductivity of each point was then corrected to the nominal temperature,  $T_{\text{nom}}$ , by use of the following equation:

$$
\lambda_{\text{corr}}(T_{\text{nom}}, \rho) = \lambda_{\text{exp}}(T_{\text{exp}}, \rho) + [\lambda(T_{\text{nom}}, \rho) - \lambda(T_{\text{exp}}, \rho)]_{\text{calc}},
$$
\n(3)

where the calculated values were obtained from the 1985/2008 IAPWS thermal-conductivity formulation.[29](#page-21-0)

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## <span id="page-5-0"></span>TABLE 1. Summary of experimental data for the thermal conductivity of water.



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<sup>a</sup>Primary data sources used in the analysis of the zero-density function are indicated by italic font.

b CC: concentric cylinders; CS: concentric spheres; DLS: dynamic light scattering; HW: heated wire; OPT: optical; PHW: pulsed heated wire; PP: parallel plates; RD: rotating disk; SHW: steady-state heated wire; SPH: spot heating; TCC: transient concentric cylinders; TG: transient grating; TH: transient heated foil; THB: thermistor bead; THS: transient hot strip; THW: transient hot-wire.

c Data derived from thermal-diffusivity measurements not included in database.

Weighted linear least-squares regression was then used to extrapolate the nominal isotherms in order to obtain the value at zero density,  $\lambda_0$ . Points were weighted with a factor equal to the inverse of the square of the estimated relative uncertainty. Both linear and quadratic density regressions were performed and the best fit was selected for  $\lambda_0$ . Confidence intervals<sup>[144](#page-23-0)</sup> constructed from the regression statistics were 95%. Some isotherms that had unusually large uncertainty bands due to inconsistencies in the underlying data were rejected from further consideration. In addition, for the isotherms of Miroshnichenko and Makhrov, $122$  the experimental uncertainty was used directly since regression was not performed on these isotherms. Also, it was not possible to perform a regression on the highest temperature isotherm, 1111.58 K, since all points were at the same pressure. In this case an average value was taken and estimated to be equivalent to the value at zero density, with an uncertainty equivalent to the experimental uncertainty.

In order to supplement the experimental dataset at very low and at high temperatures where data are unavailable or sparse, we incorporated selected theoretical data points from the recent work of Hellmann et  $al$ <sup>[145](#page-23-0)</sup> We first adjusted the theoretical values by increasing their magnitude by 1% (as recommended by the authors), $146$  and ascribed to the theoretical values an uncertainty compatible with that of the experimental data, namely, 2% (at a 2σ confidence level) for points below 800 K and 3% for points at higher temperatures. Six theoretical points covering 250–300 K and six hightemperature theoretical data points covering 800–1100 K were added to the  $\lambda_0$  dataset. In addition, in agreement with a decision made at the IAPWS meeting in 2009, the uncertainties of the experimental values were increased so that the theoretical values were within the uncertainty band.

We selected a form of equation similar to that used previously $49$  for the dimensionless thermal conductivity in the

<span id="page-7-0"></span>

FIG. 2. Dataset for  $\lambda_0$  used in the regression.

limit of zero density,

$$
\overline{\lambda}_0(\overline{T}) = \frac{\sqrt{\overline{T}}}{\sum_{k=0}^4 \frac{L_k}{\overline{T}^k}}.
$$
\n(4)

We explored weighted least squares and weighted orthogonal distance regression<sup>147</sup> for 3, 4, and 5-term expressions of the form of Eq.  $(4)$  with integer and half-integer exponents,  $k$ . Preliminary regressions identified statistical outliers, $148$  which were then excluded from the final regression. The final set of  $\lambda_0$ values contained 86 data points from 250 K to 1100 K and is shown in Fig. 2. The coefficients obtained from the regression are given in Table 2. The weights were equal to the inverse of the square of the estimated uncertainty. Figure 3 displays the percent deviation  $(100(\lambda_{0,\exp}-\lambda_{0,\text{calc}})/\lambda_{0,\text{calc}})$  between the  $\lambda_0$ data and both the  $1985/2008$  IAPWS Formulation<sup>[29](#page-21-0)</sup> and the new proposed correlation. Due to the lack of available experimental data at low and high temperatures, the 1985/2008 IAPWS Formulation shows deviations up to almost 5% at 250 K and about 6% at 2000 K from the theoretical values of Hellmann *et al.*<sup>[145](#page-23-0)</sup> The present formulation attempts to address this deficiency by incorporating theoretical values to guide the extrapolation behavior of the correlation in regions where experimental data are unavailable or sparse. Figure 4 shows the present equation, the experimental  $\lambda_0$  data, the 1985/2008 IAPWS Formulation,<sup>[29](#page-21-0)</sup> and also the recent theoretical  $\lambda_0$ values from the work of Hellmann *et al.*<sup>[145](#page-23-0)</sup> that have been adjusted upward by 1%, as mentioned earlier. The theoretically calculated values were obtained in the rigid-rotor approximation with the CC-pol intermolecular potential-energy







FIG. 3. Comparisons of  $\lambda_0$  correlations with the theoretical and experimental data.

surface by Bukowski et al.<sup>[149–151](#page-23-0)</sup> and the classical trajectory method. The proposed correlation represents both the experimental and theoretical values within the experimental uncertainty of 3% in its range of validity of temperatures up to 1173 K. Although there are data up to 1191 K, we limit the upper temperature on the range of validity to be consistent with the viscosity formulation<sup>[5](#page-21-0)</sup> used in the calculation of the enhancement of thermal conductivity discussed in Sec. [3.3](#page-8-0).





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<span id="page-8-0"></span>The extrapolation to higher temperatures (2000 K) agrees with the theoretically predicted behavior to within about 3%.

#### 3.2. Residual contribution

The second factor  $\overline{\lambda}_1$  of the product in Eq. [\(2\)](#page-4-0) is the residual thermal conductivity and represents the contribution to thermal conductivity due to increasing density. This term is sometimes referred to in the literature as the excess thermal conductivity, but here we follow the alternative nomenclature of residual thermal conductivity. The critical region is not considered here; it will be treated separately in Sec. 3.3. We adopt the same general form for  $\overline{\lambda}_1$  as in an earlier work,<sup>[19](#page-21-0)</sup>

$$
\overline{\lambda}_1(\overline{T}, \overline{\rho}) = \exp\left[\overline{\rho} \sum_{i=0}^4 \left(\frac{1}{\overline{T}} - 1\right)^i \sum_{j=0}^5 L_{ij} (\overline{\rho} - 1)^j\right],\qquad(5)
$$

with coefficients  $L_{ij}$  to be determined by regression of experimental data and the dimensionless density as defined in Eq.[\(1\)](#page-4-0).

All points were initially assigned weights  $1/u^2$ , where u is the estimated relative experimental uncertainty. The uncertainties are given in Table [1](#page-5-0); they are those given by Assael et  $al$ ,<sup>[32](#page-21-0)</sup> or the original author's recommendation when available. All densities were computed with IAPWS-95. Also, any points that exhibited differences of more than three standard deviations from preliminary regressions were considered outliers and were not used in further regression analysis. In order to extend the behavior of the correlation to higher pressures, we derived thermal conductivities from the thermal-diffusivity data of Abramson $45$  and included these data in our regressions even though the data are not included in the experimental database. The thermal conductivity is related to the thermal diffusivity D with the relationship  $\lambda = D\rho c_p$ , where the density and heat capacity are obtained from the equation of state. The Abramson data are listed in Table 3, and contain points at pressures as high as 3.7 GPa, which is outside of the range of applicability of the equation of state; $\lambda^2$  $\lambda^2$  however, we include these data to guide the extrapolation behavior of the correlation at extreme pressures. The 1985/2008 IAPWS Formula-tion<sup>[29](#page-21-0)</sup> has very unrealistic behavior in this region (for example, at 800 K and 2 GPa the thermal conductivity from the 1985/ 2008 IAPWS Formulation<sup>[29](#page-21-0)</sup> is too large by more than 2 orders of magnitude) and it was desired to improve upon this behavior.

Equation (5) contains a maximum of 30 empirical terms; there is no theoretical motivation for the form or the total number of terms necessary, or which terms will best represent the experimental data. We used the orthogonal distance regression package ODRPACK (Ref. [147\)](#page-23-0) and eliminated statistically insignificant terms as indicated by ODRPACK. Some datasets received additional weight in order to represent them to near, or within, their experimental uncertainty. We also required the correlation to reproduce the value  $(0.6065 \text{ W m}^{-1} \text{ K}^{-1})$  at 298.15 K and 0.1 MPa recommended by Ramires et al.<sup>[152](#page-23-0)</sup> The critical region was represented by a

TABLE 3. Very-high-pressure thermal-conductivity data derived from the work of Abramson et al.<sup>45</sup>

T(K)	p(MPa)	$\lambda$ (mW $\mathrm{m}^{-1}\,\mathrm{K}^{-1})$
295.15	1278	866.05
296.15	0.1	609.12
296.15	0.1	600.78
296.15	720	829.45
296.15	1098.9	861.09
297.15	450	764.1
298.15	140	673.85
298.15	270	722.07
298.15	720	821.83
383.15	722.23	932.64
383.15	894.91	989.59
383.15	1724.1	1167.4
384.15	1113.5	1047.5
384.15	2046	1221.5
469.15	1080	1038
469.15	1390	1142
469.15	1450	1152.4
469.15	2220	1318.5
469.15	2250	1363.1
470.15	3699.8	1594.5
521.15	3546.8	1603.4
571.15	3548.4	1628.1
572.15	550.77	921.26
572.15	856.94	1058.6
673.15	578.51	882.06
673.15	1113.9	1110.7
673.15	2015.7	1331.5
673.15	2829.8	1519.9
673.15	3413.5	1641.2

theoretical formulation<sup>[153](#page-23-0)</sup> that contains a single parameter  $\overline{q}_D^{-1}$ , as is discussed in Sec. 3.3. A residual-term formulation with a total of 28 terms provided the best representation of the data with the fewest number of terms. The final values of the coefficients for the residual function are given in Table [4](#page-9-0). A detailed comparison of the correlation with experimental data will be presented in Sec. [4.](#page-13-0)

#### 3.3. Critical region

To represent the critical region term  $\bar{\lambda}_2$  in Eq. [\(2\)](#page-4-0), we use a simplified crossover model developed by Olchowy and Sen-gers,<sup>[153](#page-23-0)</sup> which has yielded a good practical representation of the thermal conductivity of many fluids in the critical region<sup>[154–162](#page-23-0)</sup>

$$
\overline{\lambda}_{2}(T,\rho) = \frac{\rho c_{p} R_{D} k_{B} T}{6\pi \mu \xi \lambda^{*}} \left\{ \overline{\Omega}(\overline{q}_{D}\xi) - \overline{\Omega}_{0}(\overline{q}_{D}\xi) \right\},\qquad(6)
$$

where the specific heat capacity at constant pressure,  $c_p(T,\rho)$ , is obtained from the equation of state,<sup>[1,2](#page-21-0)</sup> and the viscosity,  $\mu(T,\rho)$ , from the IAPWS standard<sup>[5,7](#page-21-0)</sup> that includes a critical enhancement term. The crossover functions  $\overline{\Omega}$  and  $\overline{\Omega}_0$  are given by

$$
\overline{\Omega}(y) = \frac{2}{\pi} \left[ \left( 1 - \kappa^{-1} \right) \arctan(y) + \kappa^{-1} y \right] \tag{7}
$$

<span id="page-9-0"></span>

and

$$
\overline{\Omega}_0(y) = \frac{2}{\pi} \left[ 1 - \exp\left(\frac{-1}{y^{-1} + y^2/3\overline{\rho}^2}\right) \right],\tag{8}
$$

where  $\kappa = c_p/c_V$  is the ratio of the isobaric and isochoric heat capacities, and where

$$
y = \overline{q}_{D}\xi(\overline{T}, \overline{\rho}).
$$
\n(9)

In these equations,  $R_D = 1.01$  is a universal dynamic amplitude ratio,  $k_B$  is the Boltzmann constant,  $\overline{q}_D$  is a cutoff wave number, and  $\xi(\overline{T}, \overline{\rho})$  is a correlation length to be specified below, so that y is a dimensionless variable. We note that the cutoff wave number  $\overline{q}_D$  appearing in the crossover function for the critical thermal-conductivity enhancement is physically similar to, but numerically different from, the cutoff wave number  $q_D$  appearing in the crossover function for the critical viscosity enhancement.[7,153,163](#page-21-0)

We find it convenient to introduce a dimensionless function Z defined by

$$
Z(y) = \frac{2}{\pi y} \left\{ \left[ (1 - \kappa^{-1}) \arctan(y) + \kappa^{-1} y \right] - \left[ 1 - \exp\left( \frac{-1}{y^{-1} + y^2 / 3\overline{\rho}^2} \right) \right] \right\}.
$$
 (10)

Equation [\(6\)](#page-8-0) for  $\overline{\lambda}_2$  can then be rewritten as

$$
\overline{\lambda}_2(T,\rho) = \frac{\rho c_p R_D k_B T \overline{q}_D}{6\pi \mu \lambda^*} Z(y). \tag{11}
$$

To express Eq. (10) in terms of dimensionless quantities, we introduce a dimensionless isobaric specific heat capacity  $\overline{c}_p$ and a dimensionless viscosity  $\overline{\mu}$ ,

$$
\overline{c}_p = \frac{c_p}{R}, \quad \overline{\mu} = \frac{\mu}{\mu^*},\tag{12}
$$

where  $R = 0.46151805 \text{ kJ kg}^{-1} \text{K}^{-1}$  is the specific gas constant as defined by the IAPWS Formulation 1995 for the Thermodynamic Properties of Ordinary Water Substance for General and Scientific Use<sup>[1](#page-21-0)</sup> and where  $\mu^* = 1 \times 10^{-6}$  Pa s as<br>defined by the LAPWS Formulation 2008 for the Viscosity of defined by the IAPWS Formulation 2008 for the Viscosity of Ordinary Water Substance.<sup>[5](#page-21-0)</sup> From Eq.  $(10)$ , it then follows that the dimensionless critical thermal-conductivity enhancement is given by

$$
\overline{\lambda}_2(\overline{T}, \overline{\rho}) = \Lambda \frac{\overline{\rho c_p T}}{\overline{\mu}} Z(y), \qquad (13)
$$

where

$$
\Lambda = \frac{\rho^* T^* R R_D k_B \overline{q}_D}{6 \pi \mu^* \lambda^*}
$$
(14)

is a numerical constant. To avoid numerical truncation issues in Eq. (10) for small values of y, the function  $Z(y)$  is subject to the condition

$$
Z(y) = 0 \quad \text{for} \quad y < 1.2 \times 10^{-7}.\tag{15}
$$

We note that the calculation of  $Z(\overline{q}_D \xi)$  from Eq. (10) may be sensitive to the order of addition/subtraction of terms, so that parentheses have been put around the terms representing the separate contributions from  $\overline{\Omega}$  and  $\overline{\Omega}_0$  to the function Z.

A procedure has already been developed for calculating the correlation length  $\xi$  for use in the formulation for the viscosity of  $H_2O^{5,7}$  $H_2O^{5,7}$  $H_2O^{5,7}$ . The same procedure can be used in the present formulation for the thermal conductivity of  $H_2O$ ,

$$
\xi = \xi_0 \left(\frac{\Delta \overline{\chi}}{\Gamma_0}\right)^{\nu/\gamma} \tag{16}
$$

in terms of  $\Delta \overline{\chi}(\geq 0)$  defined by

$$
\Delta \overline{\chi}(\overline{T}, \overline{\rho}) = \overline{\rho} \left[ \left( \frac{\partial \overline{\rho}(\overline{T}, \overline{\rho})}{\partial \overline{p}} \right)_{\overline{T}} - \left( \frac{\partial \overline{\rho}(\overline{T}_{\mathbb{R}}, \overline{\rho})}{\partial \overline{p}} \right)_{\overline{T}} \frac{\overline{T}_{\mathbb{R}}}{\overline{T}} \right] \tag{17}
$$

or

$$
\Delta \overline{\chi}(\overline{T}, \overline{\rho}) = \overline{\rho} \bigg[ \varsigma(\overline{T}, \overline{\rho}) - \varsigma(\overline{T}_{\text{R}}, \overline{\rho}) \frac{\overline{T}_{\text{R}}}{\overline{T}} \bigg],\tag{18}
$$

with

$$
\varsigma = \left(\frac{\partial \overline{\rho}}{\partial \overline{p}}\right)_{\overline{T}}.\tag{19}
$$

In these equations,  $v = 0.630$  and  $\gamma = 1.239$  are critical exponents,  $\xi_0 = 0.13$  nm and  $\Gamma_0 = 0.06$  are critical amplitudes, and  $\overline{T}_R = 1.5$  is a reduced reference temperature. When  $\Delta \overline{\chi}$ calculated from Eq. (17) is less than zero, it must be set to zero for calculations to proceed.

For general and scientific use, the specific heat capacity  $\overline{c}_p$  in Eq. (13), the ratio  $\kappa$  of the specific heat capacities in Eq. (9) for the function Z, and the derivatives  $\left(\partial \overline{\rho}/\partial \overline{p}\right)_{\overline{T}}$  in Eq. (17) are to be calculated from the IAPWS-95 formulation for the thermo-dynamic properties<sup>[1](#page-21-0)</sup> and the viscosity  $\overline{\mu}$  in Eq. (13) from the <span id="page-10-0"></span>IAPWS Formulation  $2008$  for the viscosity.<sup>[5](#page-21-0)</sup> For industrial use, the specific heat capacity  $\overline{c}_p$  in Eq. [\(13\)](#page-9-0), the ratio  $\kappa$  of the specific heat capacities in Eq. [\(10\)](#page-9-0) for the function Z, and the derivatives  $\left(\partial \overline{\rho}/\partial \overline{p}\right)_{\overline{T}}$  in Eq. [\(17\)](#page-9-0) may be calculated from IAPWS-IF97 (Ref. [3](#page-21-0)) and the viscosity  $\overline{\mu}$  in Eq. [\(13\)](#page-9-0) may be approximated by the background viscosity  $\overline{\mu}_b$  without the critical viscosity enhancement.<sup>[5](#page-21-0)</sup> Here, we only consider the results when Eq. [\(13\)](#page-9-0) for  $\overline{\lambda}_2(\overline{T}, \overline{\rho})$  is calculated in conjunction with IAPWS-95. $<sup>1</sup>$  $<sup>1</sup>$  $<sup>1</sup>$  Further discussion of the formulation for</sup> industrial use is provided in Sec. [5.](#page-19-0)

Equation [\(13\)](#page-9-0) completely specifies the critical enhancement term  $\overline{\lambda}_2(\overline{T}, \overline{\rho})$  except for the selection of a suitable value for the universal dynamic amplitude ratio  $R<sub>D</sub>$  and the system-dependent cutoff wave number  $\overline{q}_D$ . The most reliable predictions for  $R_D$  obtained from the dynamic renormalization group theory seem to be the values  $R_D = 1.038$  found by Paladin and Peliti<sup>[164,165](#page-23-0)</sup> and  $R_D$  = 1.063 found by Folk and Mozer.<sup>[166](#page-23-0)</sup> The mode-coupling theory yields in first approximation  $R_D$  =  $1.00$ ,<sup>[167](#page-23-0)</sup> but when memory and non-local effects are included, one obtains an improved estimate of  $R_D = 1.03$ .<sup>[168](#page-23-0)</sup> Hence, Luettmer-Strathmann et al.<sup>[169,170](#page-23-0)</sup> adopted as a probable theoretical estimate  $R_D = 1.05 \pm 0.03$ . Experimental values for  $R_D$ have been reviewed by Privman  $e\hat{t}$  al.<sup>[171](#page-23-0)</sup> and by Sengers  $et al.<sup>170,172</sup>$  $et al.<sup>170,172</sup>$  $et al.<sup>170,172</sup>$  The experimental values appear to be consistent with a value of  $R_D$  slightly larger but close to unity. In a previous publication, Sengers et al.<sup>[170](#page-23-0)</sup> have reported an attempt to determine an optimum value for  $R_D$  from measurements of the decay rate of the critical fluctuations obtained by Desmarest and co-workers for  $H_2O$ .<sup>[48,173–175](#page-21-0)</sup> For this purpose, an estimate was needed for the value of the background thermal conductivity  $\lambda_b$  at the critical density  $\rho = \rho_c$  which was obtained from the 1985/2008 IAPWS Formulation for the Thermal Conductivity of  $H_2O$  (Ref. [29\)](#page-21-0) available at that time. It was then concluded that the dynamic light-scattering data were consistent with  $R_D = 1.05 \pm 0.05$ . However, a new analysis of the experimental thermal-conductivity data has yielded an estimate for  $\lambda_b$  substantially larger than the one implied by the  $1985/2008$  IAPWS Formulation<sup>[29](#page-21-0)</sup> for the Thermal Conductivity. Hence, we have repeated the analysis with the new background  $\lambda_{\rm b}$ ,

$$
\lambda_{\mathsf{b}} = \lambda^* \times \overline{\lambda}_0(\overline{T}, \overline{\rho}) \times \overline{\lambda}_1(\overline{T}, \overline{\rho}). \tag{20}
$$

This re-analysis $170$  of the light-scattering data yielded  $R_D = 1.01 \pm 0.05$ , which turns out to be close to the experimental value  $R_D = 1.00 \pm 0.04$  obtained by Wilkinson *et al.* for xenon.[176](#page-23-0) Hence, in the present formulation for the thermal conductivity of H<sub>2</sub>O we have adopted  $R_D = 1.01$  to determine  $\Lambda$ in Eq. [\(14\)](#page-9-0). We note that the initial conclusions in Ref. [170](#page-23-0) were based on use of the 1985/2008 IAPWS Formulation for the Thermal Conductivity for  $\lambda_{\rm b}$  and not on a background thermal conductivity proposed by Le Neindre et al.,<sup>[15](#page-21-0)</sup> as was erroneously stated in Ref. [170.](#page-23-0)

The cutoff wave number was determined by an iterative process. Starting with an estimate for  $\overline{q}_D$ , the critical enhancement  $\overline{\lambda}_2(\overline{T}, \overline{\rho})$  was calculated and subtracted from experimental thermal-conductivity data in the critical region to get an

TABLE 5. Critical-region constants.

Constant	Value
	177.8514
$\frac{\Lambda}{\overline{q}_{\mathrm{D}}^{-1}}$	$0.40$ nm
$\mathcal{V}$	0.630
γ	1.239
	$0.13$ nm
	0.06
$\frac{\xi_0}{\Gamma_0}$ $\frac{\overline{T}_R}{\overline{T}_R}$	1.5

improved estimate of the background contribution. This process was repeated until a consistent representation of the thermal-conductivity data in the critical region was obtained with  $\overline{q}_D^{-1} = 0.4$  nm. This value for  $\overline{q}_D^{-1}$  is similar to those found for some other simple fluids.<sup>[153](#page-23-0)</sup> The values of the constants needed to compute the critical enhancement  $\overline{\lambda}_2(\overline{T}, \overline{\rho})$  from Eq. [\(13\)](#page-9-0) for  $H_2O$  are summarized in Table 5. Note that the value of Λ in Table 5 should be used in the formulation, rather than computing  $Λ$  from Eq. [\(14\).](#page-9-0)

There are three sources of detailed experimental information for  $H_2O$  in the critical region: thermal-conductivity data obtained by Sirota et al.<sup>[14,37–44](#page-21-0)</sup> with a parallel-plate appara-tus,<sup>[177](#page-23-0)</sup> thermal-conductivity data obtained by Le Neindre and co-workers<sup>[15,124](#page-21-0)</sup> with a coaxial-cylinder apparatus,<sup>[177](#page-23-0)</sup> and dynamic light-scattering measurements for the decay rate of the critical fluctuations obtained by Desmarest and co-workers.[48,173–175](#page-21-0)

Dynamic light scattering yields values for the thermal diffusivity  $D(q)$  at a finite wave number q. A procedure for converting the experimental measurements for  $D(q)$  into values for the thermal diffusivity  $D$  in the hydrodynamic limit  $q = 0$  has been presented in a previous publication.<sup>[170](#page-23-0)</sup> In implementing the procedure, an estimate for the background thermal conductivity  $\lambda_b$  deduced from the 1985/2008 IAPWS Formulation<sup>[29](#page-21-0)</sup> was adopted, as mentioned earlier. For the present project, the procedure was repeated but with the background thermal conductivity  $\lambda_b$  calculated from Eq. (20). The values thus deduced from the light-scattering data reported by Desmarest et al.<sup>[48](#page-21-0)</sup> for the thermal diffusivity D as a function of  $\Delta T = T - T_c$  are presented in Table [6](#page-11-0) (which is identical to Table 1 in the Erratum of Ref. [170\)](#page-23-0), together with the corresponding thermal-conductivity values  $\lambda = \rho c_p D$  with  $c_p$  calculated from IAPWS-95.<sup>[1](#page-21-0)</sup> On comparing this information with the values listed in Table 1 of Ref. [170,](#page-23-0) one sees that the values deduced for D and, hence, for  $\lambda$ , from the lightscattering measurements are insensitive to the estimates adopted for the background thermal conductivity. Thus we adopt the thermal-conductivity values in Table [6](#page-11-0) as primary experimental data for the thermal conductivity of  $H_2O$  close to the critical temperature.

A plot of these thermal-conductivity data as a function of  $\Delta T$  $= T - T_c$  is presented in Fig. [5.](#page-11-0) The curve in this figure represents the values calculated from the present formulation, Eq. [\(2\)](#page-4-0). The present formulation reproduces these experimental conductivities with a standard deviation of 3%, which is within the experimental accuracy of  $3\% - 5\%$  in D and  $\pm 5$  mK in  $\Delta T$ .<sup>[48,175](#page-21-0)</sup>

<span id="page-11-0"></span>TABLE 6. Thermal diffusivity  $D(q)$  at  $\rho = \rho_c$  measured at  $q = 1.549 \times 10^7 \text{ m}^{-1}$ by Desmarest *et al.*<sup>[48](#page-21-0)</sup> and the corresponding values for  $D = D(0)$  and  $\lambda$  as a function of  $\Delta T = T - T_c$ .

$\Delta T$ (K)	$D(q)$ $(m^2 s^{-1})$	$D(m^2 s^{-1})$	$\lambda = \rho c_p D$ $(W m^{-1} K^{-1})$	$\mu/\mu_b$
0.020	$2.79 \times 10^{-10}$	$1.84 \times 10^{-10}$	4.320	1.18
0.036	$3.28 \times 10^{-10}$	$2.52 \times 10^{-10}$	3.266	1.16
0.077	$4.36 \times 10^{-10}$	$3.80 \times 10^{-10}$	2.278	1.13
0.106	$4.79 \times 10^{-10}$	$4.33 \times 10^{-10}$	1.868	1.12
0.118	$5.28 \times 10^{-10}$	$4.82 \times 10^{-10}$	1.863	1.12
0.159	$6.26 \times 10^{-10}$	$5.84 \times 10^{-10}$	1.661	1.11
0.245	$8.29 \times 10^{-10}$	$7.92 \times 10^{-10}$	1.437	1.09
0.286	$9.39 \times 10^{-10}$	$9.04 \times 10^{-10}$	1.392	1.09
0.351	$10.61 \times 10^{-10}$	$10.28 \times 10^{-10}$	1.275	1.08
0.438	$11.73 \times 10^{-10}$	$11.44 \times 10^{-10}$	1.120	1.07
0.522	$13.10 \times 10^{-10}$	$12.83 \times 10^{-10}$	1.039	1.07
0.550	$13.52 \times 10^{-10}$	$13.26 \times 10^{-10}$	1.014	1.07
0.668	$15.61 \times 10^{-10}$	$15.36 \times 10^{-10}$	0.950	1.06
0.740	$16.58 \times 10^{-10}$	$16.34 \times 10^{-10}$	0.903	1.06
0.877	$18.62 \times 10^{-10}$	$18.40 \times 10^{-10}$	0.841	1.05

Since the critical enhancement of the viscosity  $\mu$  is only significant in a very small range of temperatures and densities around the critical point, $7,163$  one often approximates the viscosity  $\mu$  in Eq. [\(11\)](#page-9-0) for the critical thermal-conductivity enhancement by the background viscosity  $\mu_{\rm b}$ .<sup>[153](#page-23-0)</sup> However, the light-scattering data provide information sufficiently close to the critical temperature where this approximation is no longer justified, as demonstrated by the values listed for  $\mu/\mu_b$  in Table 6. Thus, for an accurate representation of the thermal conductivity very close to the critical point, the full viscosity  $\overline{\mu}$ is to be kept in Eq. [\(13\).](#page-9-0)

Experimental data for the thermal conductivity in the critical region were reported by Sirota in a series of eight papers during the period from 1973 to  $1981$ .<sup>14,38-44</sup> The thermal-conductivity data were obtained with a parallel-plate apparatus.[177](#page-23-0) Articles in the series discuss various corrections applied to the original measurements. Our analysis is based on a summary table of the final data provided by Sirota for use by



FIG. 5. Thermal conductivity  $\lambda$  as a function  $\Delta T = T - T_c$  at  $\rho = \rho_c$  very close to the critical temperature. The symbols represent the thermal-conductivity data deduced from the dynamic light-scattering measurements as listed in Table 6. The curve represents the values calculated from the present formulation.

IAPWS.[37](#page-21-0) The experimental data are along isobars as a function of temperature on IPTS-48. The temperatures have been converted to ITS-90 and the densities have been recalculated according to IAPWS-95. $<sup>1</sup>$  $<sup>1</sup>$  $<sup>1</sup>$ </sup>

Le Neindre et al.<sup>[15,124](#page-21-0)</sup> published two sets of experimental thermal-conductivity data obtained with a coaxial-cylinder apparatus.<sup>[177](#page-23-0)</sup> While the data of Sirota et  $al$ .<sup>[37](#page-21-0)</sup> were obtained along isobars as a function of temperature, the data of Le Neindre et al. were obtained along isotherms as a function of pressure. The temperatures reported by Le Neindre et al. in 1973 (Ref. [15](#page-21-0)) are based on IPTS-48, while the data reported in 1987 (Ref. [124](#page-22-0)) are based on IPTS-68. Again all temperatures were converted to ITS-90 (Ref. [32\)](#page-21-0) and the densities were recalculated according to the IAPWS Formulation 1995 for the thermodynamic properties.<sup>[1](#page-21-0)</sup>

The work of Tufeu and Le Neindre<sup>[124](#page-22-0)</sup> represents an extensive effort to improve upon the experimental information earlier obtained by Le Neindre  $et al.<sup>15</sup>$  $et al.<sup>15</sup>$  $et al.<sup>15</sup>$  in the same laboratory. The authors state that the two datasets agree within their respective 2% claimed accuracy. They also appear to be consistent with the dynamic light-scattering data for the thermal diffusivity[.175](#page-23-0) The thermal-conductivity data of Sirota et al. deviate from the data obtained by Le Neindre and coworkers by as much as 10% in the density range from 200 to 400 kg m<sup> $-3$ </sup> at supercritical temperatures up to 673 K.<sup>[124](#page-22-0)</sup> On the 22.6 MPa isobar, there are deviations amounting to 25%.<sup>[175](#page-23-0)</sup> Moreover, as we shall see below, the data of Sirota et al. exhibit some unphysical behavior regarding the location of the thermal-conductivity maxima in the critical region. Hence, we have adopted the experimental data of Tufeu and Le Neindre, $124$  together with the dynamic light-scattering data of Desmarest et  $al$ ,  $48$  as the primary experimental information to validate the present thermal-conductivity formulation for describing the thermal conductivity of  $H_2O$  in the critical region.

The thermal conductivity of  $H_2O$  at supercritical temperatures is shown as function of the density in Fig. [6.](#page-12-0) The symbols indicate the experimental data reported by Tufeu and Le Neindre.<sup>[124](#page-22-0)</sup> The curves represent the values calculated from the present formulation. There are some deviations at  $T = 652$  K, which is only 5 K above  $T_c$ , where it becomes difficult to measure the thermal conductivity with the coaxialcylinder measurement technique very accurately. Moreover, close to the critical temperature the effects of increased uncertainty in the density on the thermal conductivity become appreciable. However, we know from Fig. 5 that the thermalconductivity formulation does reproduce the correct limiting behavior near the critical temperature. This is further demonstrated in Fig. [7](#page-12-0), which includes the thermal conductivity as a function of density at temperatures very close to  $T_c$ . Hence, we conclude that the formulation does yield a satisfactory description of the critical enhancement consistent with the available experimental accuracy.

The experimental thermal conductivity data reported by Sirota *et al.*<sup>[37](#page-21-0)</sup> are shown in Fig. [8](#page-12-0). A comparison of the data of Sirota et al.<sup>[37](#page-21-0)</sup> in Fig. [8](#page-12-0) with those of Tufeu and Le Neindre<sup>[124](#page-22-0)</sup> in Fig. [6](#page-12-0) reveals some considerable inconsistencies between the two datasets. While the two datasets show similar magnitudes

<span id="page-12-0"></span>

FIG. 6. Thermal conductivity  $\lambda$  as a function of the density  $\rho$  at supercritical temperatures. The symbols indicate the experimental data reported by Tufeu and Le Neindre.<sup>124</sup> The curves represent values calculated from the present formulation.

for the thermal conductivity, the density dependence of the thermal-conductivity data of Sirota *et al.*<sup>[37](#page-21-0)</sup> appears to be unphysical, displaying a maximum at some isobars well above the critical density  $\rho = \rho_c$ . This problem was already noticed in the development of the previous IAPWS formulation for the thermal conductivity of  $H<sub>2</sub>O$ , when the densities were calculated from the reported pressures and temperatures by using other equations of state.<sup>[19](#page-21-0)</sup> In the critical region, the density becomes very sensitive to small errors in either the pressure or the temperature. While Sirota et  $al^{37}$  $al^{37}$  $al^{37}$  put great efforts in determining the thermal conductivity, we conclude that either the reported pressures or temperatures are not sufficiently accurate to deduce reliable values for the corresponding densities.



FIG. 7. Thermal conductivity  $\lambda$  as a function of the density  $\rho$  at temperatures very close to  $T_c$ . The solid triangles indicate the thermal-conductivity data deduced from the dynamic light-scattering measurements of Desmarest et al.<sup>[48](#page-21-0)</sup> Additional symbols show the experimental data reported by Tufeu and Le Neindre[.124](#page-22-0) The curves represent values calculated from the present formulation.



FIG. 8. Thermal conductivity  $\lambda$  as a function of the density  $\rho$  at constant pressures. The symbols indicate experimental data reported by Sirota et al.<sup>3</sup> The curves represent values calculated from the present formulation.

#### 3.4. Computer-program verification

Tables 7 and [8](#page-13-0) are provided to assist the user in computerprogram verification. The thermal-conductivity values are calculated as a function of the tabulated temperatures and densities. Regarding Table 7, we note that some derivatives from IAPWS-95 (Ref. [1\)](#page-21-0) diverge at  $\rho = 0$ ; for those points in Table 7 $\overline{\lambda}_2$  must be set to zero. For the liquid points at 298.15 K,  $\Delta \overline{\chi}$  calculated from Eq. [\(17\)](#page-9-0) is less than zero, so (as stated in Sec. [3.3](#page-8-0)) it must be set to zero for calculations to proceed and  $\overline{\lambda}_2 = 0.$ 

#### 3.5. Liquid  $H<sub>2</sub>O$  at 0.1 MPa

It is useful to have simplified correlating equations for the properties of liquid water at atmospheric pressure with uncertainties no greater than those of the more complex formulations that cover wide ranges of temperature and pressure. For this purpose, IAPWS has recommended formulations for the properties, including thermal conductivity, of liquid water as a function of absolute temperature  $T$  at a standard pressure of 0.1 MPa.<sup>[178](#page-23-0)</sup> Details on these correlations are presented in Ref. [178](#page-23-0); we present only the thermal-conductivity correlation here. An older recommended correlation for the thermal conductivity of liquid water at 0.1 MPa is provided in Ref. [179](#page-23-0); Eq. [\(21\)](#page-13-0) below replaces that correlation.

The dimensionless thermal conductivity  $\overline{\lambda}$  of liquid water at a pressure of 0.1 MPa is described by the following

TABLE 7. Sample points for computer-program verification of the correlating equation, Eq. [\(2\).](#page-4-0) At these points,  $\overline{\lambda}_2 = 0$ .

T(K)	$\rho$ (kg m <sup>-3</sup> )	$\lambda$ (mW m <sup>-1</sup> K <sup>-1</sup> )
298.15	$\theta$	18.434 188 3
298.15	998	607.712868
298.15	1200	799.038 144
873.15	$\theta$	79.103 465 9

<span id="page-13-0"></span>TABLE 8. Sample points for computer-program verification of the correlating equation, Eq. [\(2\),](#page-4-0) including the critical-enhancement contribution  $\overline{\lambda}_2$ . For all points,  $\overline{\lambda}_0(647.35 \text{ K}) = 51.5764797$ .

T(K)	$\rho$ (kg m <sup>-3</sup> )	$\lambda_1$	$\bar{\lambda}$	$\lambda$ (mW m <sup>-1</sup> K <sup>-1</sup> )
647.35		1.006 849 7	0.0001300	51.929 892 4
647.35	122.	2.144 517 3	20.316 232 0	130.922885
647.35	222	3.484 073 6	188.091 206	367.787459
647.35	272	4.223 370 8	540.133 176	757.959776
647.35	322	4.968 1953	1187.51354	1443.75556
647.35	372	5.696 1250	356.53333	650.319402
647.35	422	6.3973429	118.931 062	448.883487
647.35	750	11.5870532	3.341 930 3	600.961346

equation:

$$
\overline{\lambda} = \sum_{i=1}^{4} c_i(\tilde{T})^{d_i},\tag{21}
$$

where  $\tilde{T} = T/(300 \text{ K})$  and  $c_i$  and  $d_i$  are coefficients and exponents given in Table 9. Equation (21) is recommended for use in the following temperature range:

$$
273.15 \text{ K} \le T \le 383.15 \text{ K} \tag{22}
$$

and extrapolates in a physically reasonable manner down to 253.15 K, as shown in Fig. 9. At some temperatures within this range, the equilibrium phase at 0.1 MPa is a vapor (above approximately 372.26 K), or is a solid (below approximately 273.15 K), and at these conditions Eq. (21) describes the thermal conductivity of a metastable phase. Equation (21) is fitted to values from Eq. [\(2\)](#page-4-0) in this work, and the uncertainty is the same as that of Eq. [\(2\)](#page-4-0) that will be discussed in Sec. [4.2](#page-16-0).



FIG. 9. Thermal conductivity of liquid water at 0.1 MPa, showing supercooled region. Experimental data have been converted from thermaldiffusivity data.

TABLE 9. Coefficients  $c_i$  and  $d_i$  in Eq. (21) for the thermal conductivity of liquid water at 0.1 MPa.

	$c_i$	$d_i$
	1.6630	$-1.15$
$\overline{c}$	$-1.7781$	$-3.4$
$\mathbf{c}$	1.1567	$-6.0$
	$-0.432115$	$-7.6$

## 4. Evaluation

In summary, the recommended formulation for the thermal conductivity is given by Eq. [\(2\),](#page-4-0)

$$
\overline{\lambda} = \overline{\lambda}_0(\overline{T}) \times \overline{\lambda}_1(\overline{T}, \overline{\rho}) + \overline{\lambda}_2(\overline{T}, \overline{\rho}).
$$

The function  $\overline{\lambda}_0(\overline{T})$  is given by Eq. [\(4\)](#page-7-0) with coefficients in Table [2.](#page-7-0) The function  $\overline{\lambda}_1(\overline{T}, \overline{\rho})$  is given by Eq. [\(6\)](#page-8-0) with coefficients in Table [4](#page-9-0). The function  $\overline{\lambda}_2(\overline{T}, \overline{\rho})$  is given by Eq. [\(13\)](#page-9-0) with the parameters presented in Table [5,](#page-10-0) thermo-dynamic properties from the IAPWS-95 release<sup>[1](#page-21-0)</sup> and the viscosity  $\overline{\mu}$  $\overline{\mu}$  $\overline{\mu}$  from the IAPWS Formulation 2008 for the Viscosity.<sup>5</sup>

#### 4.1. Comparisons with experimental data and previous IAPWS formulation for thermal conductivity

In order to evaluate performance, we have compared the results of the new formulation, Eq. [\(2\)](#page-4-0), as well as the previous IAPWS formulation for the thermal conductivity, $29$  with the experimental database. Comparisons with all sources in the experimental database are presented in Table [10](#page-14-0), which gives the number of data points, estimated uncertainty of the data, average percent deviation, average absolute percent deviation, and the standard deviation of each data source. Some points are extrapolations of the previous IAPWS correlation, $^{29}$  $^{29}$  $^{29}$  because they are outside of the recommended range of temperatures and densities. We define the percent deviation as  $P = 100^*$  $(\lambda_{\exp}-\lambda_{\text{calc}})/\lambda_{\exp}$ , where  $\lambda_{\exp}$  is the experimental value of the thermal conductivity, and  $\lambda_{\text{calc}}$  is the value calculated from the present correlation, Eq. [\(2\).](#page-4-0) The average absolute percent deviation (AAD) is found with the expression AAD =  $(\Sigma |P|)/n$ , where the summation is over all *n* points; the average percent deviation (bias) is AVG =  $(\Sigma P)/n$ , and the standard deviation is STDEV =  $([n\Sigma P^2 - (\Sigma P)^2]/n^2)^{1/2}$ . As indicated in Table [10,](#page-14-0) the results of the new formulation and the previous IAPWS formulation for the thermal conductiv- $ity^{29}$  $ity^{29}$  $ity^{29}$  are in most cases comparable, with significant differences observed in only a few cases, especially at very high pressures, as discussed below.

Selected graphical comparisons of the new correlation and the  $1985/2008$  IAPWS Formulation<sup>[29](#page-21-0)</sup> are shown in Figs. [10–14.](#page-16-0) Figures [10a](#page-16-0) and [10b](#page-16-0) show the percent deviation of selected high-pressure experimental data from the model proposed here, as well as from the 1985/2008 IAPWS Formulation. In the very-high-pressure region, above 500 MPa,

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TABLE 10. Summary of comparisons of Eq. [\(2\)](#page-4-0) with experimental data and of the previous IAPWS formulation for the thermal conductivity with experimental data.



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there are only four sets of data available: Amirkhanov et  $al$ ,  $^{119}$  $^{119}$  $^{119}$ Lawson *et al.*, <sup>[62](#page-21-0)</sup> the very old data of Bridgman<sup>[50](#page-21-0)</sup> and the more recent work of Abramson et  $al.^{45}$  $al.^{45}$  $al.^{45}$  The 1985/2008 IAPWS Formulation<sup>[29](#page-21-0)</sup> was not designed to be used in this region, and deviations are quite large as is evident from Fig. [10b](#page-16-0). Since the equation of state is valid up to 1 GPa, we included the highpressure data in our regression in order to improve the behavior of the thermal-conductivity formulation at high pressures. Comparisons with the data show that the deviations are within 5% up to 1 GPa, and within 15% to 4 GPa. Figures [11a](#page-16-0) and [11b](#page-16-0) emphasize the region between 100 MPa and 500 MPa. In this region, we felt that the most reliable data are those of Dietz et al.<sup>[117](#page-22-0)</sup> covering the temperature range of 303 K–523 K at pressures up to 350 MPa with an estimated uncertainty of 1%. The data of Lawson et  $al$ .<sup>[62](#page-21-0)</sup> and the 1979 data of Amirkhanov et al.<sup>[106](#page-22-0)</sup> are consistent with those of Dietz. It had been noted earlier by Sengers *et al.*<sup>[19](#page-21-0)</sup> that some of the other data from the Amirkhanov group appear to be inconsistent with the data of other investigators, and this is evident in the deviation plots. Finally, we present Fig. [12](#page-16-0) to illustrate how the high-pressure behavior of the

proposed equation extrapolates well to high pressures and corrects deficiencies in this region of the 1985/2008 IAPWS Formulation.[29](#page-21-0)

Figures [13a](#page-17-0) and [13b](#page-17-0) show deviations for liquid water at atmospheric pressure. Deviations from the recommended data of Ramires et al.<sup>[152](#page-23-0)</sup> are also displayed. The recommended data have an uncertainty of 0.7%. Both the proposed correlation and the  $1985/2008$  IAPWS Formulation<sup>[29](#page-21-0)</sup> represent the recommended data to within their estimated uncertainty, with the exception of one point at 370 K where the deviation from the 1985/2008 IAPWS Formulation<sup>[29](#page-21-0)</sup> slightly exceeds  $0.7\%$ . Figures [14a](#page-17-0) and [14b](#page-17-0) display deviations for water vapor at low pressures. The behavior of both formulations is similar in this region, although there are some differences for specific datasets. The  $1985/2008$  IAPWS Formulation<sup>[29](#page-21-0)</sup> represents the dataset of Keyes and Vines<sup>[71](#page-22-0)</sup> better, while the present formulation yields a better representation of the dataset of Tarzimanov et  $al$ <sup>[67](#page-21-0)</sup> Figures [15a](#page-18-0) and [15b](#page-18-0) show deviations for supercritical water for both the present model and the 1985/  $2008$  IAPWS Formulation.<sup>[29](#page-21-0)</sup> Again, the performance is

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<span id="page-16-0"></span>

FIG. 10. (a) Percent deviation as a function of pressure for selected highpressure data, present correlation. (b) Percent deviation as a function of pressure for selected high-pressure data, 1985/2008 IAPWS Formulation.<sup>25</sup>

comparable. The present formulation has a small negative bias in the very-high-temperature region compared to the dataset of Vargaftik et  $al^{72}$  $al^{72}$  $al^{72}$  This is due to the formulation of the zerodensity limit having been made compatible with the theoretical values of Hellmann et al.,  $^{145}$  $^{145}$  $^{145}$  as was described in Sec. [3.1.](#page-4-0)

## 4.2. Range and uncertainty estimates for the correlation

The range of validity of Eq. [\(2\)](#page-4-0) for the thermal conductivity is

$$
0 < p < p_{\text{t}} \text{ and } 273.16 \le T \le 1173.15 \text{ K},
$$
\n
$$
p_{\text{t}} \le p \le 100 \text{ MPa and } T_{\text{m}}(p) \le T \le 1173.15 \text{ K},
$$
\n
$$
100 \text{ MPa} < p \le 250 \text{ MPa for } T_{\text{m}}(p) \le T \le 874 \text{ K},
$$
\n
$$
250 \text{ MPa} < p \le 687 \text{ MPa for } T_{\text{m}}(p) \le T \le 573 \text{ K},
$$
\n
$$
687 \text{ MPa} < p \le 785 \text{ MPa for } T_{\text{m}}(p) \le T \le 403 \text{ K},
$$
\n
$$
785 \text{ MPa} < p \le 1000 \text{ MPa for } T_{\text{m}}(p) \le T \le 348 \text{ K},
$$



FIG. 11. (a) Percent deviation as a function of pressure for selected highpressure data, present correlation. (b) Percent deviation as a function of pressure for selected high-pressure data, 1985/2008 IAPWS Formulation.<sup>[29](#page-21-0)</sup>



FIG. 12. Selected high-pressure isotherms of the proposed correlation and the 1985/2008 IAPWS Formulation.<sup>[29](#page-21-0)</sup>

<span id="page-17-0"></span>

FIG. 13. (a) Comparisons for liquid water at atmospheric pressure, present correlation. (b) Comparisons for liquid water at atmospheric pressure, 1985/ 2008 IAPWS Formulation.<sup>29</sup>

where  $T_{\rm m}$  is the pressure-dependent melting temperature and  $p_{\rm t}$ is the triple-point pressure; both are given in Refs. [180](#page-23-0) and [181](#page-23-0). In addition, based on comparisons with the data of Abramson et al.,<sup>[45](#page-21-0)</sup> Eq. [\(2\)](#page-4-0) provides reasonable extrapolation behavior for pressures to 4 GPa for temperatures up to 673 K. This is outside the range of applicability of the equation of state<sup>[2](#page-21-0)</sup> (that only extends to 1 GPa), but both the densities and the thermal conductivities show physically realistic behavior. The 1985/ 2008 IAPWS Formulation<sup>[29](#page-21-0)</sup> is not valid in this region at all as it displays physically unrealistic behavior. In addition, at high temperatures and low densities, Eq. [\(2\)](#page-4-0) extrapolates in a physically reasonable manner above 1173.15 K, as noted in Sec. 2.1. However, the extrapolation will become increasingly unrealistic above approximately 1400 K, where a contribution to the thermal conductivity due to the dissociation of  $H_2O$ becomes significant. Kessel'man and Blank<sup>[182](#page-23-0)</sup> show that the thermal conductivity of water with the dissociation effect included at 1400 K and 0.1 MPa is approximately 20% greater than that of non-dissociated water, and recommendations are provided for the thermal conductivity of water with the dissociation effect included at temperatures up to  $6000 \text{ K}$ .<sup>[183](#page-23-0)</sup> For vapor states at temperatures below the triple-point temperature of 273.16 K and pressures less than



FIG. 14. (a) Comparisons for water vapor at low pressures, present work. (b) Comparisons for water vapor at low pressures, 1985/2008 IAPWS Formulation.<sup>[29](#page-21-0)</sup>

or equal to the sublimation pressure, the calculation of the thermal conductivity is dominated by the dilute-gas term, and this behaves in a physically reasonable manner down to at least 250 K. For the metastable subcooled liquid at atmospheric pressure, Eq. [\(2\)](#page-4-0) behaves in a physically reasonable manner down to 250 K, as discussed further in Sec. [4.3](#page-18-0). Finally, for stable fluid states outside the range of validity of Eq. [\(2\)](#page-4-0), but within the range of validity of IAPWS-95,<sup>[1](#page-21-0)</sup> the extrapolation behavior of Eq. [\(2\)](#page-4-0) is physically reasonable.

The critical thermal-conductivity enhancement is significant in a large range of temperatures and densities. This is illustrated in Fig. [16](#page-18-0), which shows contours in the temperaturedensity plane encompassing regions around the critical point where the contribution from the critical enhancement term  $\overline{\lambda}_2(\overline{T}, \overline{\rho})$  to the total thermal conductivity  $\overline{\lambda}(\overline{T}, \overline{\rho})$  exceeds 5%, 1%, 0.5%, and 0.1%. The information in this figure can be used as a guide to decide at which temperatures and densities the critical-enhancement term needs to be included to calculate the thermal conductivity with a given accuracy. We note that the range of the critical thermal-conductivity enhancement for H2O is comparable to that observed for other fluids such as carbon dioxide.<sup>[184](#page-23-0)</sup>

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<span id="page-18-0"></span>

FIG. 15. (a) Comparisons for supercritical water, present work. (b) Comparisons for supercritical water, 1985/2008 IAPWS Formulation.<sup>29</sup>

**600 700 800 900 1000 1100 1200**

*T* **/ K**

**-15**

For estimates of uncertainty outside the critical region, we relied upon comparisons with the 1985/2008 IAPWS Formu- $\frac{1}{2}$  in regions where it is applicable, and also with a subset of the experimental data. The uncertainty estimates are sum-



FIG. 16. Contours in the temperature-density plane where the contribution from the critical-enhancement  $\lambda_2$  to the total thermal conductivity  $\lambda$  equals 5%, 1%, 0.5%, and 0.1%, respectively.



FIG. 17. Estimated uncertainties for Eq. [\(2\)](#page-4-0).

marized in Fig. 17 and can be considered as estimates of a combined expanded uncertainty with a coverage factor  $k = 2$ . Finally, the present formulation reproduces the recommended value<sup>[152](#page-23-0)</sup> of the thermal conductivity at 298.15 K and 0.1 MPa,  $0.6065 \text{ W m}^{-1} \text{ K}^{-1}$ , to within the number of significant figures provided, and also represents all recommended values in Ref. [152](#page-23-0) from 275 K to 370 K at 0.1 MPa to within the stated uncertainty of 0.7%.

#### 4.3. Supercooled region

The thermal diffusivity of supercooled water has been measured by Benchikh et  $al^{47}$  $al^{47}$  $al^{47}$  and by Taschin et  $al^{46}$  $al^{46}$  $al^{46}$  These data were converted to thermal conductivity with the density and heat capacity calculated from the recommended equations in Ref. [178](#page-23-0) for supercooled water. Figure [9](#page-13-0) shows the agreement between the correlating Eq. [\(2\)](#page-4-0) and these two datasets, demonstrating that the extrapolation behavior of Eq. [\(2\)](#page-4-0) into the supercooled liquid region is reasonable. For comparison purposes, the extrapolation behaviors of the 1985/2008 IAPWS formulation for the thermal conductivity<sup>[29](#page-21-0)</sup> and of the compact equation for 0.1 MPa, Eq. [\(21\),](#page-13-0) are also shown. Equations [\(2\)](#page-4-0) and [\(21\)](#page-13-0) are indistinguishable on the plot.

The isobaric heat capacity  $c_p$  of supercooled water exhibits an anomalous increase at temperatures below 270 K as measured by Angell et  $al$ .<sup>[185](#page-23-0)</sup> and Archer and Carter.<sup>[186](#page-23-0)</sup> The IAPWS-95 formulation in the region of supercooled water was fitted to the  $c_p$  measurements of Angell et al. The subsequent  $c_p$  measurements of Archer and Carter are believed to have a slightly higher accuracy.<sup>[187](#page-23-0)</sup> Conversion of the thermaldiffusivity data to thermal-conductivity data with the aid of an equation that reproduces the  $c_p$  measurements of Archer and Carter yields slightly lower values of thermal conductivity, but differ by no more than 1% from the values obtained using  $c_p$ from IAPWS-95, which is within the experimental accuracy of the data. $188$ 

Tarzimanov, 1989



#### <span id="page-19-0"></span>TABLE 11. Coefficients  $A_{ii}$  in Eq. (26) for  $\zeta_1(\overline{T}_R, \overline{\rho})$ .

## 5. Recommendations for Industrial Applications

## 5.1. Industrial application of the correlating equation

The recommended formulation for calculating the thermal conductivity in conjunction with IAPWS-IF97 (Ref. [3](#page-21-0)) has a form similar to Eq. [\(2\)](#page-4-0),

$$
\overline{\lambda}_1(\overline{T}, \overline{\rho}) = \overline{\lambda}_0(\overline{T}) \times \overline{\lambda}_1(\overline{T}, \overline{\rho}) + \overline{\lambda}_{21}(\overline{T}, \overline{\rho}), \qquad (24)
$$

where the functions  $\overline{\lambda}_0(\overline{T})$  and  $\overline{\lambda}_1(\overline{T}, \overline{\rho})$  are identical to those<br>specified in Eqs. (4) and (5) but where for the industrial specified in Eqs. [\(4\)](#page-7-0) and [\(5\),](#page-8-0) but where for the industrial application we use

$$
\overline{\lambda}_{2I}(\overline{T}, \overline{\rho}) = \Lambda \frac{\overline{\rho c_p T}}{\overline{\mu}} Z(\overline{q}_D \xi).
$$
 (25)

In Eq. (25), the isobaric specific heat capacity  $\overline{c}_p$ , as well as the density derivatives in Eq. [\(17\)](#page-9-0) for specifying the correlation length  $\xi$  in Eqs. [\(13\)](#page-9-0) and [\(16\)](#page-9-0) for the function Z, are to be calculated from IAPWS-IF97.<sup>[3](#page-21-0)</sup> The function  $\zeta(\overline{T}_{\text{R}}, \overline{\rho})$  in Eq. [\(19\)](#page-9-0) is for industrial applications to be calculated from

$$
\zeta_{\rm I}(\overline{T}_{\rm R}, \overline{\rho}) = \frac{1}{\sum_{i=0}^{5} A_{ij} \overline{\rho}^{i}} \tag{26}
$$

with coefficients  $A_{ij}$  given in Table 11. The dimensionless viscosity,  $\overline{\mu}$ , in Eq. (25) should be calculated from the recommended viscosity correlation for industrial application as described in the IAPWS Formulation 2008 for the Viscosity of Ordinary Water Substance.<sup>[5](#page-21-0)</sup> If one needs the thermal conductivity for an industrial application as a function of pressure, rather than as a function of density, the pressures should be converted to densities to be entered into Eqs. (24) and (25) by using the IAPWS Industrial Formulation 1997 for the thermodynamic properties.<sup>[3](#page-21-0)</sup>

The subscript *j* denotes ranges defined as

$$
j = 0: \overline{\rho} \le 0.310\ 559\ 006,
$$
  
\n
$$
j = 1: 0.310\ 559\ 006 < \overline{\rho} \le 0.776\ 397\ 516,
$$
  
\n
$$
j = 2: 0.776\ 397\ 516 < \overline{\rho} \le 1.242\ 236\ 025,
$$
  
\n
$$
j = 3: 1.242\ 236\ 025 < \overline{\rho} \le 1.863\ 354\ 037,
$$
  
\n
$$
j = 4: 1.863\ 354\ 037 < \overline{\rho}.
$$

#### 5.2. Range of validity of the industrial equation

The range of validity for the industrial application of the thermal conductivity correlation, Eq. (24), is

$$
p \le 100 \text{ MPa for } 273.15 \text{ K} \le T \le 1073.15 \text{ K},
$$
  
\n
$$
p \le 50 \text{ MPa for } 1073.15 \text{ K} \le T \le 1173.15 \text{ K}.
$$
 (28)

TABLE 12. Program-verification table for  $\lambda(p,T)$  in Region 1 for the industrial application of the correlating equation.

$p$ (MPa) T(K)	20 620	50 620
$\lambda$ (mW m <sup>-1</sup> K <sup>-1</sup> )	$0.481485195 \times 10^3$	$0.545038940 \times 10^3$
$\overline{\lambda}_0$	$0.484911627 \times 10^{2}$	$0.484911627 \times 10^{2}$
$\overline{\lambda}_1$	$0.966869008 \times 10$	$0.111212177 \times 10^{2}$
$\overline{\lambda}$	$0.126391714 \times 10^{2}$	$0.575816285 \times 10$
$\rho$ (kg m <sup>-3</sup> )	$0.613227777 \times 10^{3}$	$0.699226043 \times 10^{3}$
$(\partial \rho / \partial p)_T$ at $\rho$ , T	$0.520937820 \times 10$	$0.184869007 \times 10$
$(\text{kg m}^3 \text{ MPa}^1)$		
$(\partial \rho/\partial p)_T$ at $\rho$ , $T_R$	0.935 037 951	0.639 306 277
$(\text{kg m}^3 \text{ MPa}^1)$		
$\zeta$ (nm)	0.377 694 973	0.189692422
$c_p$ (kJ kg <sup>-1</sup> K <sup>-1</sup> )	$0.763433705 \times 10$	$0.532047725 \times 10$
$c_V$ (kJ kg <sup>-1</sup> K <sup>-1</sup> )	$0.303793441 \times 10$	$0.291692653 \times 10$
Z(y)	0.166 942 638	0.113 592 223
$\mu$ ( $\mu$ Pa s)	$0.709051068 \times 10^{2}$	$0.841527945 \times 10^{2}$

<span id="page-20-0"></span>TABLE 13. Program-verification table for  $\lambda(p,T)$  in Region 2 for the industrial application of the correlating equation.

$p$ (MPa) T(K)	0.3 650	50 800
$\lambda$ (mW m <sup>-1</sup> K <sup>-1</sup> )	$0.522311024 \times 10^{2}$	$0.177799914 \times 10^3$
$\lambda_0$	$0.518787461 \times 10^{2}$	$0.698329394 \times 10^{2}$
$\overline{\lambda}_1$	$0.100678943 \times 10$	$0.244965343 \times 10$
$\bar{\lambda}$	$0.129246457 \times 10^{-3}$	$0.664341394 \times 10$
$\rho$ (kg m <sup>-3</sup> )	$0.100452141 \times 10$	$0.218\,030\,012\times 10^3$
$(\partial \rho / \partial p)_T$ at $\rho$ , T $(\text{kg m}^3 \text{ MPa}^1)$	$0.336351419 \times 10$	$0.661484493 \times 10$
$(\partial \rho/\partial p)_T$ at $\rho$ , $T_R$ $(\text{kg m}^3 \text{ MPa}^1)$	$0.223819386 \times 10$	$0.312182530 \times 10$
$\xi$ (nm)	$0.104305448 \times 10^{-2}$	0.193 491 903
$c_p$ (kJ kg <sup>-1</sup> K <sup>-1</sup> )	$0.207010035 \times 10$	$0.590718707 \times 10$
$c_V$ (kJ kg <sup>-1</sup> K <sup>-1</sup> )	$0.159675313 \times 10$	$0.252343426 \times 10$
Z(y)	$0.121437275 \times 10^{-2}$	0.137 263 826
$\mu$ ( $\mu$ Pa s)	$0.234877453 \times 10^{2}$	$0.393727534 \times 10^{2}$

#### 5.3. Estimated uncertainty of the industrial equation

The uncertainty of the industrial equatio

n results from two contributions: (1) the uncertainty of the recommended correlating equation for general and scientific use, illustrated in Fig. [17,](#page-18-0) and (2) the deviation caused by using the industrial formulation for the thermodynamic properties, industrial equation for the viscosity, and the approximation for the compressibility at the reference temperature  $T_R$ . Since the latter is much smaller than the former, the uncertainties shown in Fig. [17](#page-18-0) are applicable to the industrial equation except for a small region near the critical point, where deviations of the industrial equation become larger.

### 5.4. Computer-program verification of the industrial equation

Tables [12–](#page-19-0)14 are provided to assist the user in computerprogram verification for industrial use. The IAPWS Industrial Formulation 1997 for the Thermodynamic Properties of Water and Steam<sup>[3](#page-21-0)</sup> exhibits unphysical behavior for  $\zeta$  and  $\overline{c}_p$  very close





to the critical point, exhibiting discontinuities and taking large positive and negative values. To avoid this problem, if the value of  $\zeta$  computed from Eq. [\(19\)](#page-9-0) with IAPWS-IF97 is negative, or if it is greater than  $1 \times 10^{13}$ ,  $\zeta$  should be set equal to  $1 \times 10^{13}$ . Similarly, if the value of  $\overline{c}_p$  computed with IAPWS-IF97 is negative, or if it is greater than  $1 \times 10^{13}$ ,  $\overline{c}_p$ should be set equal to  $1 \times 10^{13}$ . In addition, due to some numerical implementations of the equation of state, the calculated singularity in the first derivative in Eq. [\(18\)](#page-9-0) may not occur exactly at  $T_c = T^*$  and  $\rho_c = \rho^*$  as it should. Therefore, calculated values of  $\overline{\lambda}_2$  may behave unphysically at points extremely close to the critical point (approximately within 0.01 kg m<sup>-3</sup> of  $\rho_c$  on the critical isotherm). The formulation should be used with caution in this region. Tables [12–](#page-19-0)14 correspond to Region 1, Region 2, and Region 3, respectively, as defined in IAPWS-IF97.<sup>[3](#page-21-0)</sup> In Region 5,  $\overline{\lambda}_2 = 0$ .

## 6. Discussion

The international task group, comprising members affiliated with IAPWS and IATP, has completed its examination of the data, theory, and models most appropriate for describing the thermal conductivity of water over broad ranges of temperature and pressure. The resulting Eq. [\(2\),](#page-4-0) with subsidiary equations and the accompanying tables of coefficients and parameters, should allow calculation of the thermal conductivity of water for most purposes according to international consensus and within uncertainty bounds achievable with current information. As evidenced by Fig. [17](#page-18-0), there are still regions (for example, at pressures above 500 MPa or temperatures above about 800 K) where new experimental data with low uncertainties could lead to improvements in future representations of the thermal-conductivity surface of water. Furthermore, improvements in theory may better elucidate the high-temperature extrapolation behavior.

The form of Eq. [\(2\)](#page-4-0) and the general forms of the constituent factors are very similar to those established in the earlier standard formulation described in Ref. [29.](#page-21-0) However, the new equation provides an improved theoretical description of the critical region, allows calculations in a broader range of state variables, considers an expanded set of experimental data, and is consistent with the more recent consensus document for the thermodynamic properties of water. The comparisons of Sec. [4](#page-13-0) provide support for the uncertainty estimates over the full range of applicability of the correlation.

The current IAPWS Release on the Thermal Conductivity of Ordinary Water Substance<sup>[6](#page-21-0)</sup> provides a concise description of the correlating equations for potential users. This paper provides a more detailed explanation of the formulation.

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## 7. References

- <sup>1</sup>IAPWS, Revised Release on the IAPWS Formulation 1995 for the Thermodynamic Properties of Ordinary Water Substance for General and Scientific Use (International Association for the Properties of Water and Steam, 2009), www.iapws.org.
- <sup>2</sup>W. Wagner and A. Pruß, [J. Phys. Chem. Ref. Data](http://dx.doi.org/10.1063/1.1461829) **31**, 387 (2002).  $31.48\%$  *Papieod Balagge on the IAPWS Industrial Egermulation 100*
- $3$ IAPWS, Revised Release on the IAPWS Industrial Formulation 1997 for Thermodynamic Properties of Water and Steam (International Association for the Properties of Water and Steam, 2007), www.iapws.org.
- 4 W. Wagner, J. R. Cooper, A. Dittman, J. Kijima, H.-J. Kretzschmar, A. Kruse, R. Mareš, K. Oguchi, H. Sato, I. Stöcker, O. Šifner, Y. Takaishi, I. Tanishita, J. Trübenbach, and T. Willkommen, [J. Eng. Gas Turbines Power](http://dx.doi.org/10.1115/1.483186) 122[, 150 \(2000\).](http://dx.doi.org/10.1115/1.483186)
- ${}^{5}$ IAPWS, Release on the IAPWS Formulation 2008 for the Viscosity of Ordinary Water Substance (International Association for the Properties of Water and Steam, 2008), www.iapws.org.
- 6 IAPWS, Release on the IAPWS Formulation 2011 for the Thermal Conductivity of Ordinary Water Substance (International Association for the Properties of Water and Steam, 2011), www.iapws.org.
- 7 M. L. Huber, R. A. Perkins, A. Laesecke, D. G. Friend, J. V. Sengers, M. J. Assael, I. N. Metaxa, E. Vogel, R. Mareš, and K. Miyagawa,[J. Phys. Chem.](http://dx.doi.org/10.1063/1.3088050) Ref. Data 38[, 101 \(2009\).](http://dx.doi.org/10.1063/1.3088050)
- 
- ${}^{8}$ J. Kestin, [J. Eng. Power](http://dx.doi.org/10.1115/1.3678145) **87**, 86 (1965).<br><sup>9</sup>J. Kestin and J. H. Whitelaw, J. Eng. Power **88**, 82 (1966).
- $^{10}$ J. V. Sengers and A. Michels, in Proceedings of the 2nd Symposium on Thermophysical Properties, edited by J. F. Masi and D. H. Tsai (American Society of Mechanical Engineers (ASME), New York, 1962),
- 
- $p_1^{11}$ A. Michels, J. V. Sengers, and P. S. van der Gulik, Physica 28[, 1216 \(1962\).](http://dx.doi.org/10.1016/0031-8914(62)90135-0)  $p_1^{12}$ Kh. I. Amirkhanov, A. P. Adamov, G. D. Gasanov, S. S. Gasanov, and K. A. Gairbekov, in Proceedings of the Abstracts of All-Union Conference on Themophysical Properties of Substances, New Schemes, and Cycle Power Plants, Odessa, 11–15 September, 1964 (Odessa Institute of Refrigerant
- Industry, Odessa, Ukraine, 1964), p. 67.<br><sup>13</sup>Kh. I. Amirkhanov and A. P. Adamov, Teploenergetika 10(9), 69
- (1963).  $14A$ . M. Sirota, V. I. Latunin, and G. M. Belyaeva, Teploenergetika 20(8), 6 (1973) [Thermal Eng.  $20(8)$ , 7 (1973)]. <sup>15</sup>B. Le Neindre, R. Tufeu, P. Bury, and J. V. Sengers, Ber. Bunsenges. Phys.
- 
- Chem. 77, 262 (1973).<br><sup>16</sup>K. Scheffler, N. Rosner, J. Straub, and U. Grigull, Brennstoff-Wärme-Kraft<br>**31**, 326 (1979).
- $^{17}\mathrm{A}$ . A. Aleksandrov, Teploenergetika 27(4), 70 (1980) [Therm. Eng. 27(4), 235 (1980)].
- <sup>18</sup>J. Kestin, in Proceedings of the 9th International Conference on the Properties of Steam, edited by J. Straub and K. Scheffler (Pergamon, Oxford, 1980), pp. 283–302.<br><sup>19</sup>J. V. Sengers, J. T. R. Watson, R. S. Basu, B. Kamgar-Parsi, and R. C.
- Hendricks, [J. Phys. Chem. Ref. Data](http://dx.doi.org/10.1063/1.555718) 13, 893 (1984). <sup>20</sup>IAPS, *The 1968 IFC Formulation for Scientific and General Use* (prepared
- by the International Formulation Committee of the 6th International Conference on the Properties of Steam) (American Society of Mechanical
- Engineers (ASME), New York, 1968). <sup>21</sup>S. L. Rivkin, A. A. Aleksandrov, and E. A. Kremenevskaya, *Thermody-*
- 
- namic Derivatives for Water and Steam (Wiley, New York, 1978).<br><sup>22</sup>J. Yata and T. Minamiyama, Bull. JSME **22**[, 1234 \(1979\).](http://dx.doi.org/10.1299/jsme1958.22.1234)<br><sup>23</sup>C. A. Meyer, R. B. McClintock, G. J. Silvestri, and R. C. Spencer, *ASME* Steam Tables, 5th ed. (American Society of Mechanical Engineers
- (ASME), New York, 1983).  $24L$ . Haar, J. S. Gallagher, and G. S. Kell, *NBS/NRC Steam Tables* (Hemisphere, Washington, DC, 1984).  $^{25}$ J. Kestin and J. V. Sengers, [J. Phys. Chem. Ref. Data](http://dx.doi.org/10.1063/1.555772) 15, 305 (1986).
- 
- <sup>26</sup>[J.](http://dx.doi.org/10.1063/1.555707) Kestin, J. V. Sengers, B. Kamgar-Parsi, and J. M. H. Levelt Sengers, J.
- [Phys. Chem. Ref. Data](http://dx.doi.org/10.1063/1.555707) 13, 175 (1984).<br><sup>27</sup>IAPS, *Release on the IAPS Formulation 1985 for the Thermal Conductivity* of Ordinary Water Substance (International Association for the Properties
- 
- of Steam, 1985).<br><sup>28</sup>H. Preston-Thomas, [Metrologia](http://dx.doi.org/10.1088/0026-1394/27/1/002) 27, 3 (1990).<br><sup>29</sup>IAPWS, *Revised Release on the IAPS Formulation 1985 for the Thermal* Conductivity of Ordinary Water Substance (International Association for
- the Properties of Water and Steam, 2008).<br><sup>30</sup>K. Scheffler, M. Rosner, and M. Reimann, *Available Input of the Thermal* Conductivity of Water Substance, Revised ed. (Institut A für Thermody-
- namik, Technische Universität, München, 1977).  $31R$ . Mareš and J. Kalova, in *Proceedings of the 15th ICPWS*, Berlin, 2008,
- see http://www.15icpws.de/papers/01\_Ind-06\_Mares.pdf. <sup>32</sup>M. J. Assael, E. Bekou, D. Giakoumakis, D. G. Friend, M. A. Killeen, J.
- Millat, and A. Nagashima, [J. Phys. Chem. Ref. Data](http://dx.doi.org/10.1063/1.556056) 29, 141 (2000). <sup>33</sup>I. M. Abdulagatov, L. A. Akhmedova-Azizova, and N. D. Azizov, [J. Chem.](http://dx.doi.org/10.1021/je0342466)
- Eng. Data 49[, 688 \(2004\).](http://dx.doi.org/10.1021/je0342466)  $34$ I. M. Abdulagatov, L. A. Akhmedova-Azizova, and N. D. Azizov, [J. Chem.](http://dx.doi.org/10.1021/je049814b)
- Eng. Data 49[, 1727 \(2004\).](http://dx.doi.org/10.1021/je049814b)<br><sup>35</sup>I. M. Abdulagatov and N. D. Azizov, [Int. J. Thermophys.](http://dx.doi.org/10.1007/s10765-005-5567-5) **26**, 593 (2005).<br><sup>36</sup>I. M. Abdulagatov and U. B. Magomedov, [High Temp. High Press.](http://dx.doi.org/10.1068/htjr097) **36**, 149
- [\(2004\).](http://dx.doi.org/10.1068/htjr097)  $^{37}$ A. M. Sirota, "Experimental data on thermal conductivity of steam obtained at the All-Union Heat Engineering Institute and published in Teploener-
- getika during 1973-1981" (personal communication to J.V. Sengers, 1985). <sup>38</sup>A. M. Sirota, V. I. Latunin, and G. M. Belyaeva, Teploenergetika **21**(10), 52
- (1974) [Therm. Eng. (USSR)  $21(10)$ , 70 (1974)].<br><sup>39</sup>A. M. Sirota, V. I. Latunin, and G. M. Belyaeva, Teploenergetika  $23(1)$ , 61
- (1976) [Therm. Eng. (USSR)  $23(1)$ , 72 (1976)].<br><sup>40</sup>A. M. Sirota, V. I. Latunin, and G. M. Belyaeva, Teploenergetika  $23(5)$ , 70
- (1976) [Therm. Eng. (USSR)  $23(5)$ , 59 (1976)].<br><sup>41</sup>A. M. Sirota, V. I. Latunin, G. M. Belyaeva, and I. I. Goldshtein, Te-<br>ploenergetika  $23(6)$ , 84 (1976) [Therm. Eng. (USSR)  $23(6)$ , 75 (1976)].
- <sup>22</sup>A. M. Sirota, V. I. Latunin, G. M. Belyaeva, and I. I. Goldshtein, Te-
- ploenergetika <sup>25</sup>(2), 21 (1978) [Therm. Eng. (USSR) <sup>25</sup>(2), 17 (1978)]. 43A. M. Sirota, V. I. Latunin, I. I. Goldshtein, and N. E. Nikolaeva, Teploenergetika **26**(9), 67 (1979) [Therm. Eng. (USSR) **26**(9), 560 (1979)].<br><sup>44</sup>A. M. Sirota, V. I. Latunin, and N. E. Nikolaeva, Teploenergetika **28**(4), 72
- 
- (1981) [Therm. Eng. (USSR) <sup>28</sup>(4), 246 (1981)]. 45E. H. Abramson, J. M. Brown, and L. J. Slutsky,[J. Chem. Phys.](http://dx.doi.org/10.1063/1.1418244) <sup>115</sup>, 10461
- [\(2001\).](http://dx.doi.org/10.1063/1.1418244)  $46A$ . Taschin, P. Bartolini, R. Eramo, and R. Torre, [Phys. Rev. E](http://dx.doi.org/10.1103/PhysRevE.74.031502) **74**, 031502
- [\(2006\).](http://dx.doi.org/10.1103/PhysRevE.74.031502)<br><sup>47</sup>O. Benchikh, D. Fournier, and A. C. Boccara, [J. Physique](http://dx.doi.org/10.1051/jphys:01985004605072700) **46**, 727 (1985).
- $^{48}P$ . Desmarest, R. Tufeu, and B. Le Neindre, in *Proceedings of the 11th* International Conference on the Properties of Water and Steam, edited by M. Píchal and O. Šifner (Hemisphere, New York, 1989), pp. 182–188. <sup>49</sup>IAPWS, *Revised Release on the IAPS Formulation 1985 for the Thermal*
- Conductivity of Ordinary Water Substance (International Association for
- 
- 
- 
- 
- 
- 
- 
- the Properties of Water and Steam, 1998).<br><sup>50</sup>P. W. Bridgman, [Proc. Am. Acad. Arts Sci.](http://dx.doi.org/10.2307/20026073) **59**, 141 (1923).<br><sup>51</sup>E. Schmidt and W. Sellschopp, [Forsch. Geb. Ingenieurwes.](http://dx.doi.org/10.1007/BF02714722) **3**, 277 (1932).<br><sup>52</sup>S. W. Milverton, Proc. R. Soc. Lo
- 
- (1946).<br>
<sup>58</sup>L. Riedel, [Chem.-Ing.-Tech.](http://dx.doi.org/10.1002/cite.330231303) **13**, 321 (1951).<br>
<sup>59</sup>L. Riedel, Chem.-Ing.-Tech. **13**, 321 (1951).<br>
<sup>60</sup>A. R. Challoner and R. W. Powell, [Proc. R. Soc. London](http://dx.doi.org/10.1098/rspa.1956.0205) **A238**, 90 (1956).<br>
<sup>61</sup>A. R. Challoner and R. W. P
- 
- 
- 
- 
- 
- 
- 
- 
- 
- 

#### <span id="page-22-0"></span>033102-22 **HUBER ET AL.**

- $^{70}$ N. B. Vargaftik and L. S. Zaitseva, Inzh.-Fiz. Zh. 6(5), 3 (1963) [J. Eng.
- Phys.  $6(5)$ , 1 (1963)].<br><sup>71</sup>F. G. Keyes and R. G. Vines, [Int. J. Heat Mass Transfer](http://dx.doi.org/10.1016/0017-9310(64)90021-3) 7, 33
- [\(1964\).](http://dx.doi.org/10.1016/0017-9310(64)90021-3)<br><sup>72</sup>N. B. Vargaftik and N. Kh. Zimina, Teploenergetika 11(12), 84 (1964)
- [Therm. Eng. (USSR)  $11(12)$ , 114 (1964)].<br><sup>73</sup>J. E. S. Venart, "The thermal conductivity of water/steam," Ph.D. thesis (University of Glasgow, Glasgow, Scotland, 1964).<br><sup>74</sup>C. E. Baker and R. S. Brokaw, [J. Chem. Phys.](http://dx.doi.org/10.1063/1.1725357) **40**, 1523 (1964).<br><sup>75</sup>J. E. S. Venart, in *Proceedings of the 3rd Symposium on Thermophysical*
- 
- Properties, edited by S. Gratch (American Society of Mechanical Engi-
- 
- neers (ASME), New York, 1965), pp. 237–245.<br><sup>76</sup>T. J. S. Brain, [Int. J. Heat Mass Transfer](http://dx.doi.org/10.1016/0017-9310(67)90133-0) **10**, 737 (1967).<br><sup>77</sup>B. Le Neindre, P. Bury, R. Tufeu, P. Johannin, and B. Vodar, Paper C-2, cited in Proceedings of the 7th International conference on the Properties of Steam, Tokyo, 1968 (American Society of Mechanical Engineers (ASME), New York, 1970), p. 231.
- $^{78}$ A. A. Tarzimanov and V. S. Lozovoi, Paper C-8, cited in Proceedings of the 7th International Conference on the Properties of Steam, Tokyo, 1968 (American Society of Mechanical Engineers (ASME), New York, 1970),
- 
- 
- p. 231.  $^{79}$ T. J. S. Brain, [J. Mech. Eng. Sci.](http://dx.doi.org/10.1243/JMES_JOUR_1969_011_048_02) 11, 392 (1969).<br><sup>80</sup>J. Bach and U. Grigull, Wä[rme Stoff](http://dx.doi.org/10.1007/BF01881222)übertrag. 3[, 44 \(1970\).](http://dx.doi.org/10.1007/BF01881222)<br><sup>81</sup>L. I. Cherneeva, Teplofiz. Svoistva Gazov, Mater. 3rd Vses. Teplofiz. Konf.<br>Svoistvam Ves
- $82$ Yu. L. Rastorguev and V. V. Pugach, Teploenergetika 17(4), 77 (1970)
- [Therm. Eng. (USSR)  $17(4)$ , 116 (1970)].<br><sup>83</sup>P. M. Stupak, A. M. Aizen, and N. G. Yampolskii, Inzh. -Fiz. Zh. 19, 74
- (1970).  $84L$ . I. Cherneeva, *Two-Phase Flows and Heat Transfer* (Moscow, 1970),
- p. 146 [Heat Transfer–Sov. Res. 3, 1 (1971)].<br><sup>85</sup>M. A. Gazdiev and Yu. L. Rastorguev, Zh. Fiz. Khim. 45, 692 (1971) [Russ.
- 
- 
- J. Phys. Chem. 45, 383 (1971)].<br><sup>86</sup>C. Papadopoulos, Chem. Ind. 33, 932 (1971).<br><sup>87</sup>A. G. Turnbull, [J. Chem. Eng. Data](http://dx.doi.org/10.1021/je60048a028) 16, 79 (1971).<br><sup>88</sup>K. M. Dijkema, J. C. Stouthart, and D. A. de Vries, Wärme- Stoffübertrag.
- 
- **5,** 72 (1972).<br><sup>89</sup>R. A. Mustafaev, Inzh.-Fiz. Zh. **22**, 850 (1972).<br><sup>90</sup>N. F. Potienko and V. A. Tsymarnyi, Izmer. Tekh. **3**, 40 (1972) [[Meas.](http://dx.doi.org/10.1007/BF00995000)
- Tech.-USSR <sup>15</sup>[, 410 \(1973\)\].](http://dx.doi.org/10.1007/BF00995000) 91A. A. Tarzimanov and M. M. Zainullin, Teploenergetika <sup>20</sup>(8), 2 (1973)
- [Therm. Eng. (USSR)  $20(8)$ , 1 (1973)].<br><sup>92</sup>N. B. Vargaftik, N. A. Vanicheva, and L. V. Yakush, Inzh.-Fiz. Zh. 25, 336
- 
- (1973).<br><sup>93</sup>V. J. Castelli and E. M. Stanley, [J. Chem. Eng. Data](http://dx.doi.org/10.1021/je60060a021) **19**, 8 (1974).<br><sup>94</sup>P. M. Shurygin, V. P. Buzovkin, and V. V. Leonov, Zavod. Lab. **40**, 820
- (1974). 95Yu. L. Rastorguev, B. A. Grigoryev, and A. M. Ishkhanov, in *Proceedings* of the 8th International Conference on the Properties of Water and Steam, edited by P. Bury, H. Perdon, and B. Vodar (Editions Européennes
- Thermiques et Industries, Paris, 1975), pp. 255–264.  $^{96}$ Kh. I. Amirkhanov, A. P. Adamov, and U. B. Magomedov, Teplofiz. Vys.
- Temp. 13, 89 (1975) [High Temp. 13, 75 (1975)]. <sup>97</sup>P. Bury, P. Johannin, B. Le Neindre, R. Tufeu, and B. Vodar, in *Proceed*ings of the 8th International Conference on the Properties of Water and Steam, edited by P. Bury, H. Perdon, and B. Vodar (Editions Européennes
- Thermiques et Industries, Paris, 1975), pp. 227–242.<br><sup>98</sup>S. Takizawa, A. Nagashima, and I. Tanishita, in *Proceedings of the 8th* International Conference on the Properties of Water and Steam, edited by P. Bury, H. Perdon, and B. Vodar (Editions Européennes Thermiques et
- Industries, Paris, 1975), pp. 245–254.<br><sup>99</sup>B. Le Neindre, P. Bury, R. Tufeu, and B. Vodar, [J. Chem. Eng. Data](http://dx.doi.org/10.1021/je60070a018) 21, 265
- [\(1976\).](http://dx.doi.org/10.1021/je60070a018) 100 $N$ . V. Tsederberg, V. N. Popov, M. P. Turin, and V. V. Tsarev, Trudy Mosk.
- Energ. Inst. 313, 32 (1976).<br>
<sup>101</sup>V. N. Popov and S. G. Dulnev, Trudy Mosk. Energ. Inst. 336, 57 (1977).
- $^{102}$ Yu. L. Rastorguev, Y. A. Ganiev, and G. A. Safronov, Inzh.-Fiz. Zh. 33, 275
- (1977). 103Kh. I. Amirkhanov, A. P. Adamov, and U. B. Magomedov, Inzh.-Fiz. Zh. <sup>34</sup>
- (2), 217 (1978) [[J. Eng. Phys. \(USSR\)](http://dx.doi.org/10.1007/BF00861229) <sup>34</sup>(2), 141 (1978)]. 104S. Takizawa, A. Nagashima, and H. Murata, [Bull. JSME](http://dx.doi.org/10.1299/jsme1958.21.273) <sup>21</sup>, 273 [\(1978\).](http://dx.doi.org/10.1299/jsme1958.21.273)
- $105A$ . A. Varchenko, in *Proceedings of the 15th Thermal Conductivity* Conference, edited by V. V. Mirkovich (Springer, New York, 1978),
- pp. 255–260.<br><sup>106</sup>Kh. I. Amirkhanov, A. P. Adamov, and U. B. Magomedov, Teplofiz. Svoistva Zhidkostei i Gazov. (Akademiia nauk SSSR, Makhachkala),
- 
- 
- 
- 
- 164 (1979).<br><sup>107</sup>L. A. Curtiss, D. J. Frurip, and M. Blander, [J. Chem. Phys.](http://dx.doi.org/10.1063/1.438628) **71**, 2703 (1979).<br><sup>108</sup>L. P. Filippov and S. N. Nefedov, Zavod. Lab. **45**, 1126 (1979).<br><sup>109</sup>V. N. Popov and M. P. Turin, Trudy Mosk. Energ. Ins
- 
- [1220 \(1979\).](http://dx.doi.org/10.1299/jsme1958.22.1220)<br><sup>112</sup>J. Yata, T. Minamiyama, and K. Kajimoto, Bull. JSME **22**[, 1227 \(1979\).](http://dx.doi.org/10.1299/jsme1958.22.1227)<br><sup>113</sup>A. Frohn and M. Westerdorf, in *Proceedings of the 9th International* Conference on the Properties of Steam, edited by J. Straub and K. Scheffler (Pergamon, New York, 1979), pp. 417–424.  $114V$ . N. Popov and S. G. Dulnev, Izv. Vyssh. Uchebn. Zaved. Pribor. 23(11),
- 
- 79 (1980). 115V. N. Popov and M. P. Turin, Teplofiz. Svoistva Veshchestv i Materialov
- **14,** 51 (1980).  $116$ J. E. S. Venart, R. C. Prasad, and D. G. Stocker, in *Proceedings of the 9th* International Conference on the Properties of Steam, edited by J. Straub and
- K. Scheffler (Pergamon, New York, 1979), pp. 392–406.<br><sup>117</sup>F. J. Dietz, J. J. de Groot, and E. U. Franck, Ber. Bunsenges. Phys. Chem. 85, 1005 (1981).<br><sup>118</sup>Y. Nagasaka and A. Nagashima, Trans. JSME. Ser. B **47** (1981).<br><sup>119</sup>Kh. I. Amirkhanov, A. P. Adamov, and U. B. Magomedov, Teplofiz.
- 
- 
- Svoistva Veshchestv v Kondensir. Sostoyanii, Makhachkala 40 (1982). <sup>120</sup>A. Alloush, W. B. Gosney, and W. A. Wakeham, [Int. J. Thermophys.](http://dx.doi.org/10.1007/BF00503318) **3**, 225
- [\(1982\).](http://dx.doi.org/10.1007/BF00503318) 121Y. Nagasaka, H. Okada, J. Suzuki, and A. Nagashima, Nihon Kikai Gakkai
- Rombunshu, B 50, 690 (1984). 122V. I. Miroshnichenko and V. V. Makhrov, Teploenergetika 31(1), 64
- (1984).  $123R$ . Tufeu, L. Denielou, and B. Le Neindre, in *Proceedings of the 10th* International Conference on the Properties of Steam, edited by V. V.
- 
- Sytchev and A. A. Aleksandrov (Mir, Moscow, 1986), Vol. 1, pp. 466–479.<br><sup>124</sup>R. Tufeu and B. Le Neindre, [Int. J. Thermophys.](http://dx.doi.org/10.1007/BF00503942) **8**, 283 (1987).<br><sup>125</sup>G. G. Guseinov, Teplofiz. Svoistv Chist. Vesh. Vod. Rast. Elek., Makhach-
- 
- kala 51 (1987).<br><sup>126</sup>W. A. Wakeham and M. Zalaf, [Fluid Phase Equilib.](http://dx.doi.org/10.1016/0378-3812(87)85022-7) **36**, 183 (1987).<br><sup>127</sup>M. J. Assael, E. Charitidou, G. P. Georgiadis, and W. A. Wakeham, Ber.
- Bunsenges. Phys. Chem. 92, 627 (1988). 128M. Zalaf, "The thermal conductivity of electrically conducting liquids,"
- Ph.D. thesis (Imperial College, London, 1988). <sup>129</sup>A. A. Tarzimanov and F. R. Gabitov, Teploenergetika **36**(7), 5 (1989)
- [Therm. Eng. (USSR) 36(7), 359 (1989)].  $130$ S. P. Venkateshan, in *Proceedings of the AIAA/ASME Thermophysics and* Heat Transfer Conference, edited by K. Vafai, M. E. Ebadian, T. Diller, M. K. Jenson, T. W. Simon, A. G. Lavine, and W. Aung (American Society of
- Mechanical Engineers (ASME), New York, 1990), Vol. 129, pp. 127–132. 131V. S. Eldarov, F. I. Mamedov, S. S. Babaev, I. I. Vakhavov, and K. O. Sambur, Thermophysical Properties and Heat Exchange in Electrolytes
- (Baku, Azneftekhim, 1991), p. 3.<br><sup>132</sup>U. Gross, Y. W. Song, and E. Hahne, [Fluid Phase Equilib.](http://dx.doi.org/10.1016/0378-3812(92)85094-O) **76**, 273 (1992).<br><sup>133</sup>M. J. Assael, L. Karagiannidis, and W. A. Wakeham, in *Proceedings of the*
- ASME Winter Annual Meeting, New Orleans, LA, 1993 (American Society of Mechanical Engineers (ASME), New York, 1993), Vol. 1, p. 120. 134M. L. V. Ramires, J. M. N. A. Fareleira, C. A. Nieto de Castro, M. Dix, and
- 
- 
- W. A. Wakeham, [Int. J. Thermophys.](http://dx.doi.org/10.1007/BF00503676) **14**, 1119 (1993). <sup>135</sup>A. Watanabe, M. Susa, and K. Nagata, J. Jpn. Inst. Met. **58**, 982 (1994). <sup>136</sup>E. B. Grigorev, "Heat conductivity of binary and ternary solutions of lanthanide salts in water," Ph.D. thesis (Institute of Geothermal Researches of the Dagestan Scientific Center of the Russian Academy of Sciences,
- Makhachkala, Dagestan, Russia, 1995).<br><sup>137</sup>H. Mensah-Brown, "Thermal conductivity of liquids," Ph.D. thesis (Imper-
- ial College, London, 1996). 138M. L. V. Ramires and C. A. Nieto de Castro, [Int. J. Thermophys.](http://dx.doi.org/10.1023/A:1006628419636) 21, 671
- [\(2000\).](http://dx.doi.org/10.1023/A:1006628419636) 139IAPWS, Release: Values of Temperature, Pressure and Density of Ordinary and Heavy Water Substances at their Respective Critical Points (International Association for the Properties of Water and Steam, 1992), www.iapws.org.
- <span id="page-23-0"></span> $140A$ . M. Sirota, V. I. Latunin, and G. M. Belyaeva, "The thermal conductivity of steam in the critical region," Report to the Special Committee of IAPS,
- <sup>141</sup>A. M. Sirota, V. I. Latunin, and G. M. Belyaeva, Teplofiz. Svoistva Gazov
- 22 (1970).  $142A$ . M. Sirota, V. I. Latunin, and G. M. Belyaeva, Teplo. Massoperenos 7,
- 
- 501 (1972).<br><sup>143</sup>M. G. Cox, Metrologia 39[, 589 \(2002\).](http://dx.doi.org/10.1088/0026-1394/39/6/10)<br><sup>144</sup>F. A. Graybill and H. K. Iyer, *Regression Analysis: Concepts and Applica*-
- tions, 1st ed. (Duxbury, Belmont, CA, 1994). 145R. Hellmann, E. Bich, E. Vogel, A. S. Dickinson, and V. Vesovic, [J. Chem.](http://dx.doi.org/10.1063/1.3158830)
- 
- Phys. 131[, 014303 \(2009\).](http://dx.doi.org/10.1063/1.3158830)<br><sup>146</sup>E. Vogel (private communication to J. V. Sengers, 2009).<br><sup>147</sup>P. T. Boggs, R. H. Byrd, J. E. Rogers, and R. B. Schnabel, ODRPACK, software for orthogonal distance regression, NISTIR 4834, National Insti-
- tute of Standards and Technology, Gaithersburg, MD, 1992.<br><sup>148</sup>J. W. Tukey, Exploratory Data Analysis (Addison-Wesley, Reading, MA,
- 1977). 149R. Bukowski, K. Szalewicz, G. C. Groenenboom, and A. van der Avoird,
- Science 315[, 1249 \(2007\).](http://dx.doi.org/10.1126/science.1136371)<br><sup>150</sup>R. Bukowski, K. Szalewicz, G. C. Groenenboom, and A. van der Avoird, [J.](http://dx.doi.org/10.1063/1.2832746)
- Chem. Phys. 128[, 094313 \(2008\).](http://dx.doi.org/10.1063/1.2832746) <sup>151</sup>R. Bukowski, K. Szalewicz, G. C. Groenenboom, and A. van der Avoird, [J.](http://dx.doi.org/10.1063/1.2832858)
- Chem. Phys. 128[, 094314 \(2008\).](http://dx.doi.org/10.1063/1.2832858) 152M. L. V. Ramires, C. A. Nieto de Castro, Y. Nagasaka, A. Nagashima, M. J.
- 
- Assael, and W. A. Wakeham, [J. Phys. Chem. Ref. Data](http://dx.doi.org/10.1063/1.555963) 24, 1377 (1995). <sup>153</sup>G. A. Olchowy and J. V. Sengers, [Int. J. Thermophys.](http://dx.doi.org/10.1007/BF01133538) **10**, 417 (1989). <sup>154</sup>K. N. Marsh, R. A. Perkins, and M. L. V. Ramires, [J. Chem. Eng. Data](http://dx.doi.org/10.1021/je010001m) 47, [932 \(2002\).](http://dx.doi.org/10.1021/je010001m)  $155R$ . A. Perkins, M. L. V. Ramires, C. A. Nieto de Castro, and L. Cusco, [J.](http://dx.doi.org/10.1021/je0101202)
- [Chem. Eng. Data](http://dx.doi.org/10.1021/je0101202) 47, 1263 (2002).<br><sup>156</sup>R. A. Perkins, [J. Chem. Eng. Data](http://dx.doi.org/10.1021/je010121u) 47, 1272 (2002).<br><sup>157</sup>M. L. Huber, A. Laesecke, and R. A. Perkins, [Ind. Eng. Chem. Res.](http://dx.doi.org/10.1021/ie0300880) 42, 3163
- 
- 
- 
- 
- 
- [\(2003\).](http://dx.doi.org/10.1021/ie0300880)<br><sup>158</sup>M. L. Huber, A. Laesecke, and R. A. Perkins, [Energy Fuels](http://dx.doi.org/10.1021/ef034109e) **18**, 968 (2004).<br><sup>159</sup>M. L. Huber and R. A. Perkins, [Fluid Phase Equilib.](http://dx.doi.org/10.1016/j.fluid.2004.10.031) **227**, 47 (2005).<br><sup>160</sup>R. A. Perkins and M. L. Huber, [J. Chem. Eng. Data](http://dx.doi.org/10.1021/je800255r) **5**
- [2120 \(2008\).](http://dx.doi.org/10.1021/je800255r)<br>
<sup>162</sup>M. [J.](http://dx.doi.org/10.1063/1.3606499) Assael, J. A. M. Assael, M. L. Huber, R. A. Perkins, and Y. Takata, J.<br>
Phys. Chem. Ref. Data **40**, 033101 (2011).
- <sup>163</sup>J. V. Sengers, R. A. Perkins, M. L. Huber, and D. G. Friend, [Int. J.](http://dx.doi.org/10.1007/s10765-008-0551-5)
- [Thermophys.](http://dx.doi.org/10.1007/s10765-008-0551-5) **30**, 374 (2009).<br><sup>164</sup>G. Paladin and L. Peliti, [J. Phys. \(Paris\), Lett.](http://dx.doi.org/10.1051/jphyslet:01984004506028900) **43**, L15 (1982).<br><sup>165</sup>G. Paladin and L. Peliti, J. Phys. (Paris), Lett. **45**, L289 (1984).<br><sup>166</sup>R. Folk and G. Moser, [Phys. Rev. Lett.](http://dx.doi.org/10.1103/PhysRevLett.75.2706)
- 
- 
- $167$ K. Kawasaki, in *Phase Transitions and Critical Phenomena*, edited by C.
- Domb and M. S. Green (Academic, New York, 1976), Vol. 5a, pp. 162–403. 168H. C. Burstyn, J. V. Sengers, J. K. Bhattacharjee, and R. A. Ferrell, [Phys.](http://dx.doi.org/10.1103/PhysRevA.28.1567)
- Rev. A 28[, 1567 \(1983\).](http://dx.doi.org/10.1103/PhysRevA.28.1567) 169J. Luettmer-Strathmann, J. V. Sengers, and G. A. Olchowy, [J. Chem. Phys.](http://dx.doi.org/10.1063/1.470718)
- **103**[, 7482 \(1995\).](http://dx.doi.org/10.1063/1.470718) 170J. V. Sengers, R. A. Perkins, M. L. Huber, and B. Le Neindre, [Int. J.](http://dx.doi.org/10.1007/s10765-009-0643-x)
- Thermophys. 30, 1453 (2009);32[, 704 \(2011\) \(Erratum\).](http://dx.doi.org/10.1007/s10765-009-0643-x) <sup>171</sup>V. Privman, P. C. Hohenberg, and A. Aharony, in *Phase Transitions and* Critical Phenomena, edited by C. Domb and J. L. Lebowitz (Academic,
- 
- New York, 1999), pp. 1–134.<br><sup>172</sup>J. V. Sengers, [Int. J. Thermophys.](http://dx.doi.org/10.1007/BF00522145) **6**, 203 (1985). <sup>173</sup>P. Desmarest, R. Tufeu, Y. Garrabos, and B. Le Neindre, [Chem. Phys. Lett.](http://dx.doi.org/10.1016/0009-2614(87)85118-7)
- <sup>142</sup>[, 336 \(1987\).](http://dx.doi.org/10.1016/0009-2614(87)85118-7) 174R. Tufeu, P. Desmarest, and B. Le Neindre, [Int. J. Thermophys.](http://dx.doi.org/10.1007/BF01133536) <sup>10</sup>, 397
- 
- [\(1989\).](http://dx.doi.org/10.1007/BF01133536)<br><sup>175</sup>P. Desmarest and R. Tufeu, [Int. J. Thermophys.](http://dx.doi.org/10.1007/BF00500558) **11**, 1035 (1990).<br><sup>176</sup>R. A. Wilkinson, G. A. Zimmerli, H. Hao, M. R. Moldover, R. F. Berg, W. L. Johnson, R. A. Ferrell, and R. W. Gammon, [Phys. Rev. E](http://dx.doi.org/10.1103/PhysRevE.57.436) 57, 436
- [\(1998\).](http://dx.doi.org/10.1103/PhysRevE.57.436) 177B. Le Neindre, R. Tufeu, and A. M. Sirota, in *Measurement of the Transport* Properties of Fluids, edited by W. A. Wakeham, A. Nagashima, and J. V.
- Sengers (Blackwell Scientific, Oxford, 1991), pp. 111–160.<br><sup>178</sup>IAPWS, Revised Supplementary Release on Properties of Liquid Water at 0.1 MPa (International Association for the Properties of Water and Steam,
- 2011), www.iapws.org.<br><sup>179</sup>J. Pátek, J. Hrubý, J. Klomfar, M. Součková, and A. H. Harvey, [J. Phys.](http://dx.doi.org/10.1063/1.3043575)
- [Chem. Ref. Data](http://dx.doi.org/10.1063/1.3043575) 38, 21 (2009).  $180$ IAPWS, Revised Release on the Pressure along the Melting and Sublimation Curves of Ordinary Water Substance (International Association for the
- Properties of Water and Steam, 2011), www.iapws.org. 181W. Wagner, T. Riethmann, R. Feistel, and A. H. Harvey, [J. Phys. Chem. Ref.](http://dx.doi.org/10.1063/1.3657937)
- Data 40[, 043103 \(2011\).](http://dx.doi.org/10.1063/1.3657937) 182P. M. Kessel'man and Y. I. Blank, in *Proceedings of the 7th International* Conference on the Properties of Water and Steam, Tokyo, No. B-11,
- 1968. 183N. B. Vargaftik, Y. K. Vinogradov, and V. S. Yargin, Handbook of Physical
- Properties of Liquids and Gases, 3rd ed. (Begell House, New York, 1996). 184J. V. Sengers, in Supercritical Fluids: Fundamentals for Applications, edited by E. Kiran and J. M. H. Levelt Sengers (Kluwer, Dordrecht, 1994),
- <sup>185</sup>C. A. Angell, M. Oguni, and W. J. Sichina, [J. Phys. Chem.](http://dx.doi.org/10.1021/j100395a032) **86**, 998 (1982).<br><sup>186</sup>D. G. Archer and R. W. Carter, [J. Phys. Chem. B](http://dx.doi.org/10.1021/jp0003914) **104**, 8563 (2000).<br><sup>187</sup>V. Holten, C. E. Bertrand, M. A. Anisimov, and J. V. Sengers, J
- 
- 
- Phys. 136[, 094507 \(2012\).](http://dx.doi.org/10.1063/1.3690497) <sup>188</sup>V. Holten (private communication to J.V. Sengers, 2012).