AP 2014–02

Chair of Applied Stochastics and Risk Management

HELMUT SCHMIDT UNIVERSITÄT
Universität der Bundeswehr Hamburg
Faculty of Economic and Social Sciences
Department of Mathematics and Statistics

Working Paper

The Fundamental Theorems of Asset Pricing and the Closed-End Fund Puzzle

Gabriel Frahm, Alexander Jonen and Rainer Schüssler

May 1, 2019

The Fundamental Theorems of Asset Pricing and the Closed-End Fund Puzzle

Gabriel Frahm

Helmut Schmidt University
Faculty of Economic and Social Sciences
Department of Mathematics and Statistics
Chair of Applied Stochastics and Risk Management
Holstenhofweg 85, D-22043 Hamburg, Germany

URL: www.hsu-hh.de/stochastik
Phone: +49 (0)40 6541-2791
E-mail: frahm@hsu-hh.de

Alexander Jonen

Helmut Schmidt University
Faculty of Economic and Social Sciences
Department of Mathematics and Statistics
Chair of Applied Stochastics and Risk Management
Holstenhofweg 85, D-22043 Hamburg, Germany

URL: www.hsu-hh.de/stochastik
Phone: +49 (0)40 6541-3007
E-mail: jonen@hsu-hh.de

Rainer Schüssler

University of Rostock
Faculty of Economic and Social Sciences
Department of Economics
Chair of Applied Economic Research
Ulmenstraße 69, D-18057 Rostock, Germany

E-mail: rainer.schuessler@uni-rostock.de

Working Paper
Please use only the latest version of the manuscript. Distribution is unlimited.

Supervised by: Prof. Dr. Gabriel Frahm
Chair of Applied Stochastics and Risk Management

URL: www.hsu-hh.de/stochastik

The Fundamental Theorems of Asset Pricing and the Closed-End Fund Puzzle

Gabriel Frahm* Alexander Jonen†
Helmut Schmidt University
Chair of Applied Stochastics and
Risk Management

Rainer Schüssler‡
University of Rostock
Chair of Applied Economic Research

May 1, 2019

Abstract

We propose a solution to the closed-end fund puzzle in financial markets without a free lunch with vanishing risk. Our results are consistent with both the time-series and the cross-sectional aspect of the closed-end fund puzzle. It turns out that a closed-end fund cannot exist if the fund manager is supposed to receive a fee although he is not able to find mispriced assets in the market. By contrast, a premium can typically be observed at the initial public offering because the fund manager has access to information that enables him to create a dominant strategy. As soon as this weak arbitrage opportunity evaporates, a premium can no longer occur. The reason why a premium quickly turns into a discount might be that the fund manager stops applying a superior trading strategy at some point in time. Another possibility is that abnormal profits are transient in a competitive financial market. In any case, when the fund manager is no longer willing or able to maintain a superior strategy, the fund must trade at a discount in order to compensate for his management fee.

Keywords: Admissibility; closed-end fund puzzle; discount; fundamental theorem of asset pricing; maximal strategy; net asset value; no arbitrage; premium.

JEL Subject Classification: G12, G14.

*E-mail: frahm@hsu-hh.de.
†E-mail: jonen@hsu-hh.de.
‡E-mail: rainer.schuessler@uni-rostock.de.

1. Motivation

Many contributions have been made over the last decades to solve one of the biggest controversies in finance: the closed-end fund puzzle (CEFP). This work provides a novel perspective and a solution to the CEFP based on the fundamental theorems of asset pricing. We focus on the information flow that is available to the fund manager and investigate its impact on price and value of the closed-end fund. This is done by preserving the neoclassical assumptions of a financial market without frictions and trading constraints. In particular, we assume that there is no free lunch with vanishing risk. Nonetheless, we are able to explain the existence of discounts and premiums under minimal conditions in a very general distribution-free framework.

A closed-end fund is a publicly traded investment company. It issues a fixed number of shares, which can be traded on a security exchange. In contrast to an open-end fund, a closed-end fund does not redeem its own shares. After some period of time, the fund can be liquidated or transformed into an open-end fund. The net asset value (NAV) of the fund usually differs from its exchange price, which is often considerably lower and sometimes higher than the NAV. If the price of the fund is lower than its NAV, we say that it trades at a discount. By contrast, if the price exceeds the NAV, we say that it trades at a premium. The fact that the price of a closed-end fund typically differs from its NAV is commonly referred to as the CEFP.

Most closed-end funds trade at a discount. A premium can often be observed at the initial public offering (IPO). After a couple of months, the premium typically diminishes and turns into a discount, which usually persists over the lifetime of the fund. Near termination, price and NAV converge to one another and the discount disappears. This phenomenon is referred to as the time-series aspect of the CEFP, whereas the observation that the majority of funds trade at a discount constitutes its cross-sectional aspect. Our results are consistent with both the time-series aspect and the cross-sectional aspect of the CEFP.

From a neoclassical perspective, the CEFP poses a challenge to the theory. The existence of discounts and premiums seems to be at odds with the fundamental no-arbitrage principle of neoclassical finance. Dimson & Minio-Kozerski (1999) point out that closed-end funds "provide contemporaneous and observable market-based rates of return for both stocks and underlying asset portfolios." In fact, closed-end funds form a rare class of companies where the difference between market price and book value can be observed from market quotes. According to Cherkes *e*t al. (2009), the discrepancy between the exchange price and the NAV of a closed-end fund poses one of the longest standing anomalies in finance. Malkiel (1977) states that this is a "seeming inconsistency with the efficient-markets hypothesis." Similarly, Ross (2005) considers the CEFP a "seeming insult to rationality and the NA principle" and calls the discounts "first cousins to a violation of the law of one price." We would like to stress that discounts and premiums are only a *seeming* violation to rationality and the no-arbitrage principle. This is due to the fact that trading strategies that try to exploit discounts or premiums are, in general, inadmissible. We will discuss this important issue later in detail.

The solutions to the CEFP proposed in the literature can be assigned either to neoclassical finance or to behavioral finance. The neoclassical branch of literature tries to explain discounts or premiums by market imperfections. For example, Berk & Stanton (2007), Chay & Trzcinka (1999), Ferguson & Leistikow (2001), Ramadorai (2012), Roenfeldt & Tuttle (1973), and Ross (2002, 2005) emphasize the trade-off between expectations about managerial performance and management fees. Barclay *e*t al. (1993) and Malkiel (1995) propose agency costs as a possible explanation. Moreover, also liquidity issues (Cherkes *e*t al., 2009, Datar, 2001), market segmentation (Chan *e*t al., 2008, Chang *e*t al., 1995), replication costs (Gemmill & Thomas, 2002, Pontiff, 1996), and taxes (Brickley *e*t al., 1991, Malkiel, 1977) are taken into consideration.

The neoclassical approaches typically presume that investors are rational. By contrast, the arguments of the behavioral school are related to investor sentiment, i.e., they assume that investors are irrational (see, e.g., Abraham *e*t al., 1993, Barberis *e*t al., 1998, Bodurtha *e*t al., 1995, de Long *e*t al., 1990, Elton *e*t al., 1998, Lee *e*t al., 1991). Loosely speaking, the behavioral branch of literature claims that discounts are the result of noise trading. According to Cherkes (2012), the investor sentiment theory is not able to explain why discounts and premiums can be observed simultaneously among closed-end funds that, otherwise, appear to be similar. Further, Ramadorai (2012) reports that sentiment-based explanations cannot be supported by data on closed hedge funds. We refer to Charrón (2009), Cherkes (2012), Dimson & Minio-Kozerski (1999) as well as Garay & Russel (1999) for a comprehensive overview of the literature.

Garay & Russel (1999) conclude that "none of the theories, either individually or collectively, provide a sufficient explanation for the pricing of closed-end funds and, therefore, the enigma continues." Similarly, Charrón (2009) sums up that the CEFP "continues to be an important issue in the long standing debate between traditional finance and behavioral finance." By contrast, Ramadorai (2012) claims that "promising solutions to this puzzle have been advanced" and refers to Berk & Stanton (2007) as well as Cherkes *e*t al. (2009). However, up to now, it seems that none of these explanations is fully accepted as a satisfactory solution to the CEFP, either from a neoclassical or a behavioral perspective. Hence, despite the plethora of studies that deal with the puzzle, the quest continues.

Our approach belongs to the neoclassical branch and focuses on managerial performance. Berk & Stanton (2007) point out that "managerial ability adds value to the fund." Similarly, Chay & Trzcinka (1999) write that "discounts and premiums of closed-end funds reflect the market's assessment of anticipated managerial performance." Accordingly, Roenfeldt & Tuttle (1973) note that

> "*[...] a discount reflects investors' expectations of a less than average risk-adjusted performance based on net asset values for these funds. Conversely, a premium reflects the expectation of superior performance based on net asset values.*"

Our theoretical findings confirm their expectation hypothesis *in principle*. The essential difference is that we refer to expectations that are based on an equivalent (local) martingale measure instead of the physical measure. The no-arbitrage approach enables us to present our theory at a very general level, without having to model the risk preferences of the market participants.

Jarrow & Protter (2019) provide a similar explanation for the CEFP, but there are important conceptual differences between their approach and ours. They concentrate on transaction costs and trading constraints, whereas our solution focuses on management fees. Further, they emphasize that price bubbles are consistent with rational behavior and we argue that the existence of discounts and premiums depends on the question of whether the fund manager applies a passive or an active strategy.[1] Our results can be considered complementary to the bubble theory.

The CEFP is often attributed to market imperfections or behavioral inconsistencies. We do not assume that the market is imperfect or that the market participants are irrational. The first fundamental theorem of asset pricing enables us to explain why premiums can occur in a financial market without free lunches with vanishing risk, whereas the third fundamental theorem of asset pricing reveals why closed-end funds typically trade at a discount. We do not have to assume that the market is complete. Thus, we neglect the second fundamental theorem of asset pricing.

A major observation of our theory is that a closed-end fund can only exist if the fund manager has access to information that enables him to find a mispriced asset, i.e., to create a dominant strategy. This means that the market must be inefficient with respect to the information flow that is used by the fund manager. We are able to explain also why a premium typically occurs at the IPO but turns into a discount in the course of time. Since we consider management fees, as well as the investors' perception of managerial ability, our work is strongly related to Berk & Stanton (2007) as well as Ross (2002, 2005). We will discuss Ross' pricing formula explicitly.

The rest of the article is organized as follows: In Section 2, we present our formal definitions and assumptions. That section contains a formal description of the financial market, the different notions of arbitrage as well as the basic theorems. Section 3 deals with closed-end funds. In that section, we discuss the IPO of a closed-end fund and its pricing in an arbitrage-free market. Section 4 represents the main part of this work. There, we distinguish between passive and active trading strategies. Moreover, for each type of strategy, we analyze the case with and without a management fee. In Section 5, we provide a practical example in order to demonstrate our theoretical findings and Section 6 concludes our work.

2. Technical Preliminaries

2.1. The Financial Market

Let $(\Omega, \mathcal{G}, \mathbf{G}, \mathbb{P})$ be a filtered probability space that satisfies the usual conditions. That is, $\mathcal{G}$ is $\mathbb{P}$-complete, the filtration $\mathbf{G} := \{\mathcal{G}_t\}_{t\geq 0}$ is right-continuous, and $\mathcal{G}_0$ contains all $\mathbb{P}$-null events. We assume that the σ-field $\mathcal{G}$ is given by $\bigvee_{t\geq 0} \mathcal{G}_t$ and, for notational convenience, we omit the subscript "$t \geq 0$" whenever the index set, i.e., $\mathbb{R}_+$, is clear from the context. Hence, we do not presume that the lifetime of the financial market is finite. By contrast, the number of assets shall be finite. The assets are infinitely divisible and there are no transaction costs or trading constraints. We choose some asset as a numéraire.[2] Let $S_t = (1, S_{1t}, \dots, S_{Nt})$ be the vector of *discounted* asset prices, where $S_{0t} \equiv 1$ denotes the discounted price of the numéraire asset. In the following, we will drop the attribute "discounted." Although our exposition is based on

[1]The precise meaning of "passive" and "active" will be clarified in the subsequent analysis.
[2]It is implicitly assumed that the price process of the numéraire is positive $\mathbb{P}$-almost surely.

discounted asset prices, the following equalities and inequalities between price and NAV of a closed-end fund hold true after changing from discounted to nominal asset prices.

All statements that are related to random quantities or stochastic processes are meant to be true $\mathbb{P}$-almost surely. For example, the equality "$X = Y$" for any two random vectors X and Y means that each component of X equals the corresponding component of Y $\mathbb{P}$-almost surely. Any inequality of the form "$X \leq Y$," "$X \geq Y$," "$X < Y$," or "$X > Y$" is to be understood in the same sense. For example, if $\{X_t\}$ is an $\mathbb{R}^n$-valued stochastic process, "$\{X_t\} \geq a$" indicates that $\{X_t\}$ is $\mathbb{P}$-almost surely bounded below by $a \in \mathbb{R}$. Further, two stochastic processes are considered identical if and only if they coincide $\mathbb{P}$-almost surely, etc.

The $\mathbb{R}^{N+1}$-valued price process $\{S_t\}$ is a positive $\mathbf{G}$-adapted locally bounded semimartingale being right-continuous and having limits from the left ("càdlàg").[3] It is implicitly understood to be an equilibrium-price process. This means that the financial market clears with S_t at every time $t \geq 0$. We assume that the limit of $\{S_t\}$ exists and denote that limit by S_∞. Moreover, we suppose that the random vector S_∞ is finite and positive. The filtration $\mathbf{G}$ can be viewed as a general flow of information evolving through time. Since $\{S_t\}$ is $\mathbf{G}$-adapted, $\mathcal{G}_t$ contains, at least, the price history at every time $t \geq 0$. The asset prices are fixed at time $t = 0$. Put another way, the σ-field generated by S_0 is trivial. Moreover, the symbol $\mathbf{F}$ denotes any subfiltration of $\mathcal{G}$ that contains the evolution of asset prices and we implicitly assume that $(\Omega, \mathcal{G}, \mathbf{F}, \mathbb{P})$ satisfies the usual conditions, too.

Every $\mathbf{F}$-predictable $\mathbb{R}^{N+1}$-valued stochastic process $\{H_t\}$ with $H_t = (H_{0t}, H_{1t}, \ldots, H_{Nt})$ that is integrable with respect to $\{S_t\}$ is said to be a trading strategy based on $\mathbf{F}$. The value of the (trading) strategy at any time $t \geq 0$ is given by

$$V_t = \sum_{i=0}^{N} H_{it} S_{it} = V_0 + \int_0^t H \, \mathrm{d}S, \tag{1}$$

where $V_0 = \sum_{i=0}^{N} H_{i0} S_{i0}$ is the initial value and $\int_0^t H \, \mathrm{d}S$ quantifies the gain of the strategy up to time $t \geq 0$.[4] Hence, V_t evolves from self-financing transactions between time 0 and t. Since the strategy is $\mathbf{F}$-predictable, H_t must be determined by some information in $\mathbf{F}$ that occurs at any time *before* t.[5] The value process, $\{V_t\}$, of a strategy based on $\mathbf{F}$ is $\mathbf{F}$-adapted. Two strategies are considered identical if and only if their value processes coincide.

2.2. No-Arbitrage Conditions

We will often refer to specific no-arbitrage conditions, i.e., no free lunch with vanishing risk (NFLVR) and no dominance (ND). The former no-arbitrage condition is well-established and goes back to Delbaen & Schachermayer (1994). The latter no-arbitrage condition is introduced by Merton (1973). According to Delbaen & Schachermayer (1994, Definition 2.7), a strategy $\{H_t\}$ is called

[3] The assumption that $\{S_t\}$ is locally bounded is quite harmless, but it should not be dispensed of (Frahm, 2018).
[4] The integral $\int_0^t H \, \mathrm{d}S$ represents a *vector* stochastic integral (Jacod, 1979, Jacod & Shiryaev, 2003, Chapter III, § 6c).
[5] The only exception is H_0, which can be determined by information that is contained in $\mathcal{F}_0$.

- a-admissible if and only if $\{\int_0^t H\,\mathrm{d}S\} \geq -a$ for a *given* number $a > 0$, but just
- admissible if and only if $\{\int_0^t H\,\mathrm{d}S\} \geq -a$ for *any* number $a > 0$.

Admissibility is a fundamental requirement of no-arbitrage theory (Delbaen & Schachermayer, 1994, 1998, Harrison & Pliska, 1981, 1983). It guarantees the existence of a finite credit line that prevents the trader from going bankrupt over time. The most prominent example of an inadmissible strategy is the doubling strategy (Harrison & Kreps, 1979).

In the following, $\int_0^\infty H\,\mathrm{d}S$ denotes the final gain of the strategy $\{H_t\}$.[6] An admissible strategy $\{H_t\}$ based on $\mathbf{F}$ that is such that

(i) $\mathbb{P}\left(\int_0^\infty H\,\mathrm{d}S \geq 0\right) = 1$ and

(ii) $\mathbb{P}\left(\int_0^\infty H\,\mathrm{d}S > 0\right) > 0$

is said to be an arbitrage based on $\mathbf{F}$. It represents the possibility to make money out of nothing without bearing any risk by using the information flow $\mathbf{F}$.

Now, consider two admissible strategies $\{G_t\}$ and $\{H_t\}$ based on $\mathbf{F}$. The strategy $\{H_t\}$ is said to dominate $\{G_t\}$ if and only if

(i) $\mathbb{P}\left(\int_0^\infty H\,\mathrm{d}S \geq \int_0^\infty G\,\mathrm{d}S\right) = 1$ and

(ii) $\mathbb{P}\left(\int_0^\infty H\,\mathrm{d}S > \int_0^\infty G\,\mathrm{d}S\right) > 0$.

An admissible strategy based on $\mathbf{F}$ is said to be maximal with respect to $\mathbf{F}$ if and only if it is not dominated by another admissible strategy based on $\mathbf{F}$ (Delbaen & Schachermayer, 1997a,b).

Further, an admissible strategy $\{H_t\}$ based on $\mathbf{F}$ is said to be dominant if and only if there exists an asset $i \in \{0, 1, \ldots, N\}$ such that

(i) $\mathbb{P}\left(\int_0^\infty H\,\mathrm{d}S \geq S_{i\infty} - S_{i0}\right) = 1$ and

(ii) $\mathbb{P}\left(\int_0^\infty H\,\mathrm{d}S > S_{i\infty} - S_{i0}\right) > 0$.

We say that there is ND based on $\mathbf{F}$ if and only if each single asset is maximal with respect to $\mathbf{F}$.[7] An asset that is *not* maximal with respect to $\mathbf{F}$ can be considered overpriced: Every investor who has access to the information flow $\mathbf{F}$ is able to apply some dominant strategy and thus, irrespective of his particular risk attitude, it cannot be optimal for him to buy and hold the dominated asset. ND implies no arbitrage (NA) but the converse is not true. Moreover, the ND condition implies that no asset can be dominated on any time interval $[s, t]$ with $0 \leq s < t < \infty$.

Let $\{a_n\}_{n\in\mathbb{N}}$ be a sequence of positive numbers that converges to 0 and $\{H_{tn}\}_{n\in\mathbb{N}}$ a sequence of a_n-admissible strategies based on $\mathbf{F}$.[8] Further, let $\int_0^\infty H_n\,\mathrm{d}S$ be the final gain of the n-th strategy. According to Theorem 3.2 in Delbaen & Schachermayer (2001), we say that there is a free lunch with vanishing risk based on $\mathbf{F}$ if and only if

[6] Later, we will see that the final gain always exists and is finite in our context.

[7] Note that a buy-and-hold strategy in each single asset is admissible, since the asset prices are positive and they are fixed at time $t = 0$.

[8] Here, the symbol $\mathbb{N}$ stands for the set of positive integers, i.e., $\mathbb{N} = \{1, 2, \ldots\}$.

(i) there exists an arbitrage based on **F** *or*

(ii) for each $n \in \mathbb{N}$, there exists a natural number $m \geq n$ such that

$$\mathbb{P}\left(\left|\int_0^\infty H_m \,\mathrm{d}S\right| > \varepsilon\right) > \delta \tag{2}$$

for some fixed numbers $\delta, \varepsilon > 0$.

The second condition *essentially* describes an arbitrage, since the maximum loss of the investor can be made arbitrarily small by choosing a sufficiently large number $n \in \mathbb{N}$. However, the first condition is indispensable because an arbitrage need not satisfy the second condition (Delbaen & Schachermayer, 2001). NFLVR implies NA but the converse is not true. NFLVR also guarantees that the final gain $\int_0^\infty H \,\mathrm{d}S$ of every admissible strategy $\{H_t\}$ exists and is finite (Delbaen & Schachermayer, 1994, Theorem 3.3). Dominant strategies and free lunches with vanishing risk can be seen as weak arbitrage opportunities. We say that there is no weak arbitrage (NWA) if and only if there is ND and NFLVR (Frahm, 2016).

2.3. Basic Theorems

All probability measures considered in this work are implicitly assumed to be *equivalent* to $\mathbb{P}$. We are often concerned with a local martingale measure or a uniformly integrable martingale measure $\mathbb{Q}$.[9] A probability measure $\mathbb{Q}$ is said to be a local martingale measure with respect to **F** if and only if $\{S_t\}$ is a local $\mathbb{Q}$-martingale with respect to **F**. The set of all local martingale measures with respect to **F** is denoted by $\mathcal{L}(\mathbf{F})$. Analogously, $\mathcal{U}(\mathbf{F})$ is the set of all probability measures that are such that $\{S_t\}$ is a uniformly integrable martingale with respect to **F**.

The aforementioned sets of probability measures satisfy the following subset properties, which will be highly important in the following analysis:

- $\mathcal{U}(\mathbf{F}) \subseteq \mathcal{L}(\mathbf{F})$,
- $\mathcal{L}(\mathbf{G}) \subseteq \mathcal{L}(\mathbf{F})$, and
- $\mathcal{U}(\mathbf{G}) \subseteq \mathcal{U}(\mathbf{F})$.

The first fundamental theorem of asset pricing (1st FTAP) is established by Delbaen & Schachermayer (1994) under the assumption that $\{S_t\}$ is locally bounded, and it is generalized to the (locally) unbounded case by Delbaen & Schachermayer (1998).

Theorem 1 (1st FTAP)**.** *There is NFLVR based on **F** if and only if* $\mathcal{L}(\mathbf{F}) \neq \emptyset$.

Proof: See Delbaen & Schachermayer (1994, Section 4). Q.E.D.

The third fundamental theorem of asset pricing (3rd FTAP) is introduced by Jarrow & Larsson (2012). It is further discussed in Jarrow (2012). Frahm (2016) extends the 3rd FTAP to financial markets with infinite lifetime, in which uniform integrability plays a crucial role.

[9] Since $\mathbb{Q}$ is equivalent to $\mathbb{P}$, the filtered probability space $(\Omega, \mathcal{G}, \mathbf{F}, \mathbb{Q})$ satisfies the usual conditions for each subfiltration **F** of **G** that contains the evolution of asset prices.

Theorem 2 (3rd FTAP). *There is NWA based on* $\boldsymbol{F}$ *if and only if* $\mathcal{U}(\boldsymbol{F}) \neq \emptyset$.

Proof: See Frahm (2016, Theorem 3.1). Q.E.D.

The second fundamental theorem of asset pricing (2nd FTAP) goes back to Harrison & Pliska (1981, 1983). We do not assume that the market is complete and so the 2nd FTAP is not considered in this work. However, it is worth emphasizing that many results that are presented here could be strengthened under the assumption that the market is complete and sensitive.[10]

The following standard result of martingale theory turns out to be very useful in our context (Jacod & Shiryaev, 2003, Chapter I, Theorem 1.39).

Theorem 3 (Doob's theorem). *Let* $\{X_t\}$ *be a supermartingale with respect to* $\boldsymbol{F}$ *such that there exists an integrable random variable* Y *with* $X_t \geq \mathbf{E}(Y \mid \mathcal{F}_t)$ *for all* $t \geq 0$.

(i) The process $\{X_t\}$ *converges to a finite limit* X_∞.

(ii) If ς *and* τ *are* $\boldsymbol{F}$*-stopping times, the random variables* X_ς *and* X_τ *are integrable. Further, we have that* $X_\varsigma \geq \mathbf{E}(X_\tau \mid \mathcal{F}_\varsigma)$ *given that* $\tau \geq \varsigma$.

(iii) The stopped process $\{X_{t\wedge\tau}\}$ *is again a supermartingale with respect to* $\boldsymbol{F}$.

Proof: See the references given by Jacod & Shiryaev (2003, p. 10). Q.E.D.

We will frequently use the Ansel-Stricker theorem, which we recall here for convenience.

Theorem 4 (Ansel-Stricker theorem). *Suppose that* $\mathcal{L}(\boldsymbol{F}) \neq \emptyset$ *and fix any* $\mathbb{Q} \in \mathcal{L}(\boldsymbol{F})$. *Let* $\{H_t\}$ *be an admissible strategy based on* $\boldsymbol{F}$. *The gain process* $\{\int_0^t H\,\mathrm{d}S\}$ *is a local* $\mathbb{Q}$*-martingale and thus a* $\mathbb{Q}$*-supermartingale with respect to* $\boldsymbol{F}$.

Proof: See Ansel & Stricker (1994, Corollar 3.5). Moreover, Fatou's Lemma implies that every local martingale that is bounded below (by a martingale) is a supermartingale. Q.E.D.

3. Closed-End Funds

3.1. Initial Public Offering

The inception of a closed-end fund consists of various steps (Hanley et al., 1994). First of all, the founders create an underwriting syndicate, whose task is to initiate the public offering. For this purpose, it fixes the number of shares and their issue price. Fixing the number and price of the shares can be used for restricting the seed capital.[11] Interested parties can subscribe for shares and the syndicate members are responsible for their distribution. Also syndicate members can act as subscribers. These steps are carried out during the so-called pre-issue phase, i.e., before time $t = 0$, which is not part of our model. Typically, also the fund manager is designated during the pre-issue phase. We neglect any fee that occurs during this phase.

[10] In a complete and sensitive market, $\mathcal{U}(\mathbf{F})$ is a singleton and we may choose the growth-optimal portfolio as a numéraire, in which case $\mathbb{P}$ is the (unique) uniformly integrable martingale measure with respect to $\mathbf{F}$ (Frahm, 2016).

[11] The rationale behind this procedure will become clear below.

The IPO happens at time $t = 0$. All shares that have not been sold in the primary market are made void. The seed capital is handed over to the fund manager, who starts to apply a trading strategy until some stopping time $\tau \in (0, \infty]$ is reached. We assume that the fund is liquidated at time τ. During the lifetime of the fund, the manager receives a fee relative to its NAV. The fee rate is constant and amounts to $\phi \geq 0$.[12] It has been negotiated in the pre-issue phase and cannot be changed afterwards. At time of liquidation, τ, the NAV is paid out to the shareholders. The fund cannot redeem shares at any time before τ.

Since all assets in the market are infinitely divisible, we can assume without loss of generality that the number of issues corresponds to 1. Let P_t be the price of the fund at any time $t \geq 0$. Further, V_t denotes its NAV without taking the management fee into account, whereas W_t represents the NAV of the fund after deducing the management fee that accrued until time t. Thus, we have that $W_t = V_t e^{-\phi t}$ for all $t \geq 0$. Further, P_0 denotes the initial price of the fund and $V_0 = W_0$ corresponds to its issue price. Due to our normalization, V_0 is also the seed capital of the fund. In the following, we call V_t the value of the fund manager's trading strategy and W_t the wealth of the fund at time $t \geq 0$.

As soon as the shares are placed, they can be traded in the aftermarket. The demand of a market participant at time $t = 0$ depends on the initial price P_0. It is determined by his available information and individual preferences. Let $f(P_0) \in \overline{\mathbb{R}}$ be the aggregate demand in the market at time $t = 0$. More precisely, $f(P_0)$ is the number of shares that all market participants are willing to own at the IPO, given that the initial price amounts to P_0. The aggregate supply in the market, i.e., the number of shares that all market participants, in fact, own at the IPO, equals 1 by definition—and remains constant until liquidation. This means that the initial price is given by the solution of $f(P_0) = 1$. We assume that $f : \mathbb{R} \to \overline{\mathbb{R}}$ is continuous and strictly decreasing, which guarantees that P_0 is uniquely determined by $P_0 = f^{-1}(1)$.

Our first proposition holds true irrespective of whether or not the market is arbitrage free. It is a simple consequence of the fact that P_0 represents an equilibrium price.

Proposition 1. *A closed-end fund cannot start at a discount, i.e., we have that* $P_0 \geq V_0$.

Proof: Suppose that $P_0 < V_0$. Since the aggregate demand function f is strictly decreasing, we have that $f(V_0) < 1$, which means that the market participants, altogether, are not willing to invest as much as V_0 at time $t = 0$. This contradicts the fact that the closed-end fund obtains the seed capital V_0. Q.E.D.

The overall situation at IPO is depicted in Figure 1.

3.2. Pricing in Arbitrage-Free Markets

From now on, we assume that there is NWA with respect to **F**. This means that it is impossible to create a free lunch with vanishing risk or a dominant strategy on the basis of **F**.[13] The latter means that there is no admissible strategy based on **F** that dominates an asset in the market—on

[12] Throughout this work, ϕ is understood to be the net expense ratio of the fund, i.e., it contains the total amount of fees involved with managing the fund after reimbursement.

[13] The same holds true for any flow of information that is contained in **F**.

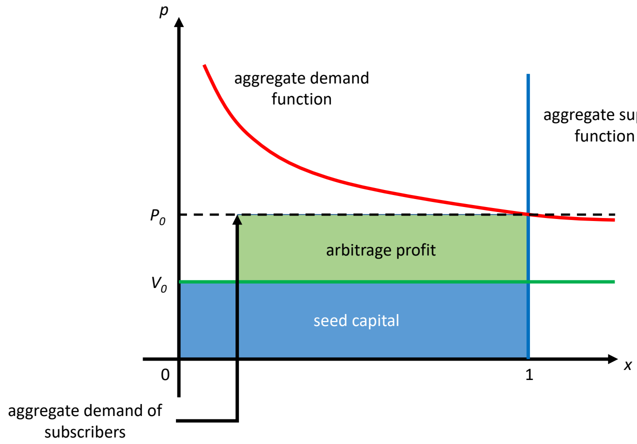

Figure 1: If the fund starts at a premium, i.e., $P_0 > V_0$, the subscribers can make an arbitrage profit by selling fund shares in the aftermarket. This is referred to as flipping (Hanley et al., 1994). As we will see later, this arbitrage opportunity can readily be justified from an economic perspective. We already mentioned that the syndicate fixes both the number of shares and the issue price. This can be done in such a way that the (expected) utility of the subscribers or, at least, of the founders is maximal—taking the arbitrage profit into account that can be made by flipping some shares. In any case, the seed capital covers at least the amount of capital that the founders are willing to invest.

any arbitrary time interval. Hence, the assets can be considered fairly priced with respect to the subfiltration **F**. The 3rd FTAP states that this situation can be characterized by the existence of a uniformly integrable martingale measure with respect to **F**.

Additionally, we assume that there is NFLVR with respect to $\mathbf{G} \supset \mathbf{F}$. The 1st FTAP tells us that this is equivalent to the existence of a local martingale measure with respect to **G**. This general setting allows us to consider situations in which the financial market is free of weak arbitrage opportunities or, at least, of free lunches with vanishing risk. However, we consider the financial market inefficient with respect to the information flow **G** if there exists a dominant strategy based on **G**, i.e., $\mathcal{U}(\mathbf{G}) = \emptyset$ (Frahm, 2016). In this case, at least one asset in the market is mispriced with respect to **G**.

Let $\{H_t\}$ be the strategy of the fund manager. We assume that $\{H_t\}$ is based on the filtration **G**, which is true whenever his strategy is based on any subfiltration **F** of **G**.[14] Further, let $\{V_t\}$ be the value process of $\{H_t\}$. Throughout this work, we presume that the initial value of the strategy, i.e., V_0, is *fixed*, which means that the σ-field generated by V_0 is trivial. Moreover, we suppose that $\{V_t\}$ is positive, which implies that $\{H_t\}$ is (V_0-)admissible. The time of liquidation, τ, is a positive and possibly infinite stopping time with respect to **G**.[15] From the time of liquidation, τ, there is no fee left and thus $W_t = W_\tau$ for all $t \geq \tau$.

The following theorem is a main pillar of our work.

Theorem 5. *Suppose that $\mathcal{L}(\mathbf{G}) \neq \emptyset$ and fix any $\mathbb{Q} \in \mathcal{L}(\mathbf{G})$. Further, let the strategy of the fund manager be based on **G**.*

[14] However, a strategy that is based on **G** need not be based on any subfiltration $\mathbf{F} \subset \mathbf{G}$.
[15] Thus, for all $t \geq 0$, one is able to decide on the basis of $\mathcal{G}_t$ whether $\tau \leq t$ or $\tau > t$.

(i) The wealth process $\{W_t\}$ is a positive supermartingale with respect to $\mathbf{G}$,

(ii) it has an integrable limit $W_\infty \geq 0$,

(iii) it holds that $W_t \geq \mathbf{E}_{\mathbb{Q}}\big(W_\tau \,|\, \mathcal{G}_t\big)$ for all $t \geq 0$, and

(iv) in the case $\phi > 0$, we have that $W_t > \mathbf{E}_{\mathbb{Q}}\big(W_\tau \,|\, \mathcal{G}_t\big)$ for all $0 \leq t < \tau$ and $W_t = W_\tau$ for all $t \geq \tau$.

Proof: (i) From Theorem 4 we conclude that the value process $\{V_t\}$ is a supermartingale with respect to $\mathbf{G}$. This means that $V_t \geq \mathbf{E}_{\mathbb{Q}}\big(V_T \,|\, \mathcal{G}_t\big)$ and thus

$$\begin{aligned} W_t &= V_t e^{-\phi t} \geq \mathbf{E}_{\mathbb{Q}}\big(V_T \,|\, \mathcal{G}_t\big) e^{-\phi t} \geq \mathbf{E}_{\mathbb{Q}}\big(V_T \,|\, \mathcal{G}_t\big) e^{-\phi T} && (3) \\ &= \mathbf{E}_{\mathbb{Q}}\big(V_T e^{-\phi T} \,|\, \mathcal{G}_t\big) = \mathbf{E}_{\mathbb{Q}}\big(W_T \,|\, \mathcal{G}_t\big) && (4) \end{aligned}$$

for all $0 \leq t \leq T < \infty$. Hence, also the wealth process $\{W_t\}$ is a supermartingale with respect to $\mathbf{G}$ and, since $V_t > 0$, it holds that $W_t = V_t e^{-\phi t} > 0$ for all $t \geq 0$. (ii–iii) By setting $Y = 0$ in Theorem 3 we conclude that the limit W_∞ exists and is integrable. Since $\{W_t\}$ is positive, we have that $W_\infty \geq 0$. Moreover, we obtain $W_t \geq \mathbf{E}_{\mathbb{Q}}\big(W_\tau \,|\, \mathcal{G}_t\big)$ for all $0 \leq t < \tau$ and, since $W_t = W_\tau$ for all $t \geq \tau$, the inequality holds true for all $t \geq 0$. (iv) Finally, in the case $\phi > 0$, we have that

$$\begin{aligned} \mathbf{E}_{\mathbb{Q}}\big(W_\tau \,|\, \mathcal{G}_t\big) &= \mathbf{E}_{\mathbb{Q}}\big(V_\tau e^{-\phi\tau} \,|\, \mathcal{G}_t\big) < \mathbf{E}_{\mathbb{Q}}\big(V_\tau e^{-\phi t} \,|\, \mathcal{G}_t\big) && (5) \\ &= \mathbf{E}_{\mathbb{Q}}\big(V_\tau \,|\, \mathcal{G}_t\big) e^{-\phi t} \leq V_t e^{-\phi t} = W_t && (6) \end{aligned}$$

for all $0 \leq t < \tau$ but $W_t = W_\tau$ for all $t \geq \tau$. Q.E.D.

We make the explicit assumption that the closed-end fund cannot go bankrupt on the long run, i.e., we have that $W_\infty > 0$ and thus also $V_\infty > 0$.

Due to the 3rd FTAP, the price process $\{P_t\}$ is a uniformly integrable martingale with respect to $\mathbf{F}$. This means that there exists a probability measure $\mathbb{Q}$ such that $P_t = \mathbf{E}_{\mathbb{Q}}\big(P_\infty \,|\, \mathcal{F}_t\big)$ for all $t \geq 0$. Technically, at time τ the fund share turns into W_τ numéraire assets and thus we have that $P_\tau = W_\tau$.[16] More precisely, it holds that $P_t = P_\tau$ for all $t \geq \tau$ and thus $P_\infty = P_\tau = W_\tau = V_\tau e^{-\phi\tau}$. To sum up, so far we have found that

(i) $P_0 \geq V_0$,

(ii) $P_t = \mathbf{E}_{\mathbb{Q}}\big(V_\tau e^{-\phi\tau} \,|\, \mathcal{F}_t\big)$ for all $0 < t < \tau$, and

(iii) $P_t = V_\tau e^{-\phi\tau}$ for all $t \geq \tau$.

We say that price and NAV are congruent if and only if $P_t = W_t$ for all $t \geq 0$. Hence, if price and NAV are congruent, discounts and premiums cannot occur during the lifetime of the fund.

The CEFP stems from the fact that price and NAV are *not* congruent in most real-life situations. In the following, we investigate the circumstances under which price and NAV must or must not differ from one another. The 1st FTAP can be used to explain the existence of premiums, whereas the 3rd FTAP enables us to explain why discounts are a typical phenomenon in competitive

[16] This argument requires only the law of one price.

financial markets. Hence, discounts and premiums need not be considered "anomalous," i.e., they are not inconsistent with the no-arbitrage principle.

In fact, the mere existence of a discount or premium does not imply an arbitrage opportunity. For example, suppose that the fund starts at a premium. Then a trader who has access to the information flow **G** could replicate the manager's strategy and enter a short position into the fund. He immediately receives $P_0 - V_0$ numéraire assets and his positions are settled when the fund is liquidated. Additionally, he even can earn the management fee in the meantime. However, selling the fund short (and holding this position until liquidation) is an *inadmissible* strategy if $\{P_t\}$ is unbounded above.

4. Passive vs. Active Trading Strategies

4.1. Passive Trading Strategy

Suppose that the manager applies a strategy based on **F**. Put another way, his strategy is based solely on information that does not allow for a weak arbitrage. In this sense, he applies a passive strategy. In contrast to the conventional terminology in finance literature, this does not mean that the fund manager aims at tracking a stock index or the like. By saying that his strategy is "passive," we just emphasize the fact that the manager does not make use of extraordinary information, i.e., of information that allows him to create a weak arbitrage.

4.1.1. Without Management Fee

In this section, we assume that $\phi = 0$, i.e., the manager does not receive any fee. Thus, we have that $W_t = V_t$ for all $t \geq 0$. As already mentioned, in the case that $P_0 > V_0$, the subscribers could flip some shares and earn an arbitrage profit. Is this possible under the given circumstances?

Theorem 6. *Suppose that $\mathcal{U}(\boldsymbol{F}) \neq \emptyset$ and fix any $\mathbb{Q} \in \mathcal{U}(\boldsymbol{F})$. If the fund manager receives no fee and his strategy is based on $\boldsymbol{F}$, then*

(i) $P_t = V_t$ for all $t \geq 0$,

(ii) the value process $\{V_t\}$ is a uniformly integrable martingale with respect to $\boldsymbol{F}$, and

(iii) the strategy of the fund manager is maximal with respect to $\boldsymbol{F}$.

Proof: (i) Since it holds that $\mathcal{U}(\mathbf{F}) \subseteq \mathcal{L}(\mathbf{F})$, we can apply the Ansel-Stricker theorem together with Theorem 3 and obtain

$$P_t = \mathbf{E}_{\mathbb{Q}}\big(P_\tau \,|\, \mathcal{F}_t\big) = \mathbf{E}_{\mathbb{Q}}\big(V_\tau \,|\, \mathcal{F}_t\big) \leq V_t \tag{7}$$

for all $0 \leq t < \tau$ and, since $P_t = V_t$ for all $t \geq \tau$, we have that $P_t \leq V_t$ for all $t \geq 0$. This implies that $P_0 \leq V_0$, but from Proposition 1 we already know that $P_0 \geq V_0$. Hence, the fund cannot start at a premium, i.e., $P_0 = V_0$. The process $\{V_t - P_t\}$ is a nonnegative supermartingale with respect to **F**. It is well-known that each nonnegative supermartingale that hits zero must stay at zero.

Hence, we have that $P_t = V_t$ for all $t \geq 0$. (ii) The price process $\{P_t\}$ is a uniformly integrable $\mathbb{Q}$-martingale with respect to **F** and, since price and value coincide, the same must hold true for the value process $\{V_t\}$. (iii) The 3rd FTAP guarantees that each asset is maximal with respect to **F**. This holds true also for the fund share and, since price and NAV are congruent, we conclude that the strategy of the fund manager is maximal with respect to **F**. Q.E.D.

Hence, if the fund manager applies a passive strategy, the fund can never trade at a discount or a premium and flipping is impossible. This confirms the common hypothesis that price and NAV must be congruent if the financial market is arbitrage free. However, it is worth emphasizing that the attribute "arbitrage free" means free of *weak* arbitrage opportunities, not only of arbitrage in the strict sense described at the beginning of Section 2.2.

What else can we say about the manager's investment policy? Our basic requirement is that the fund must not go bankrupt. This rules out any suicide strategy (Harrison & Pliska, 1981). Further, his strategy must be such that the value process $\{V_t\}$ is a martingale and not a strict supermartingale. Hence, the risk-neutral expectation regarding any future value of the fund must never fall below its present value. That is, the average performance of the manager must not be worse than the average performance of each other asset in the market—in terms of a risk-neutral measure $\mathbb{Q} \in \mathcal{U}(\mathbf{F})$ and with respect to the given filtration **F**. This places relatively low demands on his investment capabilities. We will come back to this point later.

Further, the value process of his trading strategy must be uniformly integrable, i.e., we must have that

$$\lim_{x\to\infty} \sup_{t\geq 0} \mathbf{E}_{\mathbb{Q}}\big(V_t \mathbb{1}_{\{V_t > x\}}\big) = 0\,. \tag{8}$$

Here, $\mathbb{1}_G$ denotes a Bernoulli variable, i.e., for each $\omega \in \Omega$, it holds that $\mathbb{1}_G(\omega) = 1$ if $\omega \in G \in \mathcal{G}$ and $\mathbb{1}_G(\omega) = 0$ else. Hence, the trading strategy must not produce extreme values during the lifetime of the fund. In economic terms we could say that the manager must not achieve tremendous profits by taking excessive risks, i.e., he must perform a moderate strategy. Finally, the fund manager must apply a strategy that is maximal with respect to the information flow **F**, which means that there must not be another passive (and admissible) strategy that dominates his strategy. Otherwise, the fund would not have been founded in the first place.

4.1.2. With Management Fee

Now, we assume that $\phi \geq 0$, i.e., we allow the fund manager to receive a fee.

Theorem 7. *Suppose that $\mathcal{U}(\boldsymbol{F}) \neq \emptyset$. If the fund manager applies a strategy based on $\boldsymbol{F}$, he cannot receive any fee, i.e., we have that $\phi = 0$.*

Proof: Fix any probability measure $\mathbb{Q} \in \mathcal{U}(\mathbf{F})$ and suppose that $\phi > 0$. We already know that $\{V_t\}$ is a supermartingale with respect to **F**. Thus, we conclude that

$$P_0 = \mathbf{E}_{\mathbb{Q}}\big(V_\tau e^{-\phi\tau} \,|\, \mathcal{F}_0\big) < \mathbf{E}_{\mathbb{Q}}\big(V_\tau \,|\, \mathcal{F}_0\big) \leq V_0. \tag{9}$$

However, we also know that $P_0 \geq V_0$ and so we obtain a contradiction. Q.E.D.

Hence, a fund manager will not be remunerated for applying a strategy that does not make use of extraordinary information, i.e., of information that allows for a weak arbitrage.

In our model, the price of the fund at any time $t \geq 0$ before liquidation depends on the risk-neutral probability distribution of the final wealth, W_τ, conditional on the information that is contained in $\mathcal{F}_t$. Hence, without loss of generality, we assume that capital gains (e.g., dividends and interest) are retained until the end of the fund's lifetime. A well-known neoclassical solution to the CEFP is presented by Ross (2002, 2005). In contrast to our model, Ross presumes that the fund continuously pays out dividends to the investors and a fee to the manager. He comes to the conclusion that

$$P_0 = \frac{\gamma}{\gamma + \phi} \cdot V_0 \,, \tag{10}$$

where $\gamma \geq 0$ denotes the dividend yield, which is assumed to be constant over time.[17] If the fund manager receives a fee, i.e., $\phi > 0$, Ross' pricing formula implies that $P_0 < V_0$. Hence, the fund must start at a *discount*, which cannot be observed in most real-life situations and contradicts our theoretical findings. Nonetheless, the pricing formula proposed by Ross might be suitable for calculating the price of a closed-end fund *after* its IPO. The conditions under which this could be done are discussed below.

4.2. Active Trading Strategy

Typically, a closed-end fund starts at a premium, i.e., we have that $P_0 > V_0$. Such a premium can be considered a remuneration of the founders, provided they have found a manager who is able to apply an active trading strategy. Put another way, he must have access to extraordinary information, i.e., to information that enables him to create a weak arbitrage. If there is no free lunch with vanishing risk, the given arbitrage opportunity must be a dominant strategy, which means that the manager is able to explore mispriced assets in the market. He is rewarded for his special capabilities (and his access to extraordinary information) by a permanent fee until the fund is liquidated. The premium at the IPO represents the value added by the founders. Each founder is able to flip a number of shares, in which case he turns some of his value added into cash.[18] Some market participants who are not able to apply the trading strategy on their own are willing to accept a premium at the IPO—and possibly afterwards.

To justify our arguments, we assume that the fund manager applies a strategy based on $\mathbf{G}$. Nonetheless, we make the minimal assumption that $\mathcal{L}(\mathbf{G}) \neq \emptyset$. Thus, it is still impossible to produce a free lunch with vanishing risk on the basis of $\mathbf{G}$. However, in the case that $\mathcal{U}(\mathbf{G}) = \emptyset$ one can create at least a dominant strategy based on $\mathbf{G}$, which implies that the financial market is inefficient with respect to the information flow $\mathbf{G}$ (Frahm, 2016, Jarrow & Larsson, 2012). Then, for each $\mathbb{Q} \in \mathcal{L}(\mathbf{G})$, it can happen, i.e., with positive probability, that $S_{it} > \mathbf{E}_{\mathbb{Q}}\big(S_{i\infty} \,|\, \mathcal{G}_t\big)$ for some asset $i \in \{1, 2, \ldots, N\}$ and time point $t \geq 0$.[19] This is called a price bubble (Jarrow & Protter, 2019). Of course, a price bubble can occur also for the fund share and so we can have that

[17] It can be shown that $P_0 = 0$ in the case that $\gamma = \phi = 0$.
[18] Here, it is assumed that each founder is a subscriber.
[19] Otherwise, some element of $\mathcal{L}(\mathbf{G})$ would be a uniformly integrable martingale measure, which means that $\mathcal{U}(\mathbf{G}) \neq \emptyset$.

$P_t > \mathbf{E}_{\mathbb{Q}}(P_\tau \,|\, \mathcal{G}_t)$ for some time point $0 \leq t < \tau$. If we ignore the management fee and suppose that the value process $\{V_t\}$ is a uniformly integrable martingale with respect to $\mathbf{G}$, we obtain

$$P_t > \mathbf{E}_{\mathbb{Q}}(P_\tau \,|\, \mathcal{G}_t) = \mathbf{E}_{\mathbb{Q}}(V_\tau \,|\, \mathcal{G}_t) = V_t,$$

which explains the existence of premiums. We will elaborate on this principle idea in more detail in the following sections.

4.2.1. Without Management Fee

We start by assuming that $\phi = 0$, i.e., that the fund manager receives no fee. Suppose that the value process $\{V_t\}$ is of class $\mathcal{D}$ (Jacod & Shiryaev, 2003, Chapter I, Definition 1.46), i.e., for some $\mathbb{Q} \in \mathcal{L}(\mathbf{G})$, we have that

$$\lim_{x\to\infty} \sup_{\kappa\in[0,\infty)} \mathbf{E}_{\mathbb{Q}}\big(V_\kappa \mathbb{1}_{\{V_\kappa > x\}}\big) = 0\,, \tag{11}$$

where κ represents a nonnegative finite $\mathbf{G}$-stopping time. Hence, the strategy of the fund manager must not lead to extreme values—at *any* finite stopping time that depends only on the information flow $\mathbf{G}$. In this case, a discount can never take place after the IPO. Similarly, if the price process $\{P_t\}$ is of class $\mathcal{D}$, it does not lead to extreme values in the precise sense described above. In that case, the investors cannot be ecstatic about the future prospects of the fund and a premium can never occur during the lifetime of the fund. This is stated, in a more general form, by the next theorem.

Theorem 8. *Suppose that $\mathcal{L}(\boldsymbol{G}) \neq \emptyset$ and fix any $\mathbb{Q} \in \mathcal{L}(\boldsymbol{G})$. Further, let the strategy of the fund manager be based on $\boldsymbol{G}$.*

(i) If the price process $\{P_t\}$ is of class $\mathcal{D}$, it is a uniformly integrable martingale with respect to $\boldsymbol{G}$ and we have that $P_t \leq V_t$ for all $t \geq 0$.

(ii) If the value process $\{V_t\}_{t\geq 0}$ is of class $\mathcal{D}$, it is a uniformly integrable martingale with respect to $\boldsymbol{G}$ and we have that $P_t \geq V_t$ for all $t \geq 0$.

Proof: (i) The price process $\{P_t\}$ is a local martingale with respect to $\mathbf{G}$ and suppose that it is of class $\mathcal{D}$. Proposition 1.47 in Jacod & Shiryaev (2003, Chapter I) implies that $\{P_t\}$ is a uniformly integrable martingale with respect to $\mathbf{G}$. From the Ansel-Stricker theorem we conclude that the value process $\{V_t\}$ is a supermartingale with respect to $\mathbf{G}$. Theorem 3 leads to

$$P_t = \mathbf{E}_{\mathbb{Q}}(P_\tau \,|\, \mathcal{G}_t) = \mathbf{E}_{\mathbb{Q}}(V_\tau \,|\, \mathcal{G}_t) \leq V_t \tag{12}$$

for all $0 \leq t < \tau$ and, since $P_t = V_t$ for all $t \geq \tau$, we have that $P_t \leq V_t$ for all $t \geq 0$. (ii) The Ansel-Stricker theorem reveals that the value process $\{V_t\}$ is a local martingale with respect to $\mathbf{G}$ and, if it is of class $\mathcal{D}$, Proposition 1.47 in Jacod & Shiryaev (2003, Chapter I) implies that $\{V_t\}$ is a uniformly integrable martingale with respect to $\mathbf{G}$. Moreover, the price process $\{P_t\}$ is a positive local martingale and thus it is also a supermartingale with respect to $\mathbf{G}$. Now, we obtain

$$P_t \geq \mathbf{E}_{\mathbb{Q}}(P_\tau \,|\, \mathcal{G}_t) = \mathbf{E}_{\mathbb{Q}}(V_\tau \,|\, \mathcal{G}_t) = V_t \tag{13}$$

for all $0 \leq t < \tau$ and thus $P_t \geq V_t$ for all $t \geq 0$. Q.E.D.

The following corollary provides sufficient conditions for the absence of discounts or premiums that might be considered more intuitive from an economic perspective.

Corollary 1. *Suppose that $\mathcal{L}(\boldsymbol{G}) \neq \emptyset$ and fix any $\mathbb{Q} \in \mathcal{L}(\boldsymbol{G})$. Further, let the strategy of the fund manager be based on $\boldsymbol{G}$.*

(i) If there exists a random variable X with $\mathbf{E}_{\mathbb{Q}}(X) < \infty$ such that $P_t \leq \mathbf{E}_{\mathbb{Q}}(X \,|\, \mathcal{G}_t)$ for all $t \geq 0$, we have that $P_t \leq V_t$ for all $t \geq 0$.

(ii) If there exists a random variable Y with $\mathbf{E}_{\mathbb{Q}}(Y) < \infty$ such that $V_t \leq \mathbf{E}_{\mathbb{Q}}(Y \,|\, \mathcal{G}_t)$ for all $t \geq 0$, we have that $P_t \geq V_t$ for all $t \geq 0$.

Proof: (i) Due to Proposition 1.47 in Jacod & Shiryaev (2003, Chapter I), each uniformly integrable martingale is a process of class $\mathcal{D}$. The process $\{\mathbf{E}_{\mathbb{Q}}(X \,|\, \mathcal{G}_t)\}$ is closed by X and so it is a uniformly integrable martingale with respect to $\mathbf{G}$, i.e., a process of class $\mathcal{D}$. Since $\{P_t\}$ is positive and bounded above by a process of class $\mathcal{D}$, it is also a process of class $\mathcal{D}$. From Theorem 8 we conclude that $P_t \leq V_t$ for all $t \geq 0$. (ii) Due to the same arguments, $\{V_t\}$ is of class $\mathcal{D}$. Once again, from Theorem 8, we conclude that $P_t \geq V_t$ for all $t \geq 0$. Q.E.D.

For example, suppose that the price process $\{P_t\}$ is bounded above by any constant $x > P_0$. This means that the future prospects of the investors regarding the fund are modest. If the fund starts at a premium, i.e., $P_0 > V_0$, replicating the manager's strategy and entering a short position into the fund would be an admissible strategy and thus an arbitrage. However, Corollary 1 guarantees that the fund cannot start at a premium under the given circumstances and so this simple arbitrage opportunity cannot exist.

We conclude that, even if the financial market does not allow for a free lunch with vanishing risk, it is possible to observe a premium, but for this to happen, two conditions must be satisfied:

(i) The fund manager must make use of extraordinary information, i.e., of information that goes beyond the information flow $\mathbf{F}$, and

(ii) the investors must be ecstatic about the future prospects of the fund, which means that its price process $\{P_t\}$ must *not* be of class $\mathcal{D}$.

This places high demands both on the manager's capabilities and on the goodwill of the investors.

Theorem 8 reveals that $\{P_t\}$ is a uniformly integrable martingale with respect to $\mathbf{G}$ if it is of class $\mathcal{D}$ and the same holds true for $\{V_t\}$. The following theorem states that the maximality of the fund share or of the strategy is a necessary and sufficient condition for the uniform integrability of $\{P_t\}$ or of $\{V_t\}$, respectively.

Theorem 9. *Suppose that $\mathcal{L}(\boldsymbol{G}) \neq \emptyset$ and let the strategy of the fund manager be based on $\boldsymbol{G}$.*

(i) The price process $\{P_t\}$ is a uniformly integrable martingale with respect to $\boldsymbol{G}$ for some $\mathbb{Q} \in \mathcal{L}(\boldsymbol{G})$ if and only if the fund share is maximal with respect to $\mathbf{G}$.

(ii) The value process $\{V_t\}$ *is a uniformly integrable martingale with respect to* $\mathbf{G}$ *for some* $\mathbb{Q} \in \mathcal{L}(\mathbf{G})$ *if and only if the strategy is maximal with respect to* $\mathbf{G}$.

Moreover, in (i) we have that $P_t \leq V_t$*, whereas in (ii) we have that* $P_t \geq V_t$ *for all* $t \geq 0$.

Proof: (i) The fund share represents an admissible buy-and-hold strategy based on $\mathbf{G}$. Due to Theorem 2.5 in Delbaen & Schachermayer (1997a) it is maximal with respect to $\mathbf{G}$ if and only if $\{P_t\}$ is a uniformly integrable martingale with respect to $\mathbf{G}$ for some $\mathbb{Q} \in \mathcal{L}(\mathbf{G})$. (ii) The same arguments reveal that the strategy of the fund manager is maximal with respect to $\mathbf{G}$ if and only if $\{V_t\}$ is a uniformly integrable martingale with respect to $\mathbf{G}$ for some $\mathbb{Q} \in \mathcal{L}(\mathbf{G})$. The remaining part follows from Corollary 1 by setting $X = P_\infty$ and $Y = V_\infty$, respectively. Q.E.D.

Thus, a discount reveals that the strategy of the fund manager is not maximal with respect to $\mathbf{G}$. Similarly, if we observe a premium, the fund share cannot be maximal, i.e., it is overpriced with respect to $\mathbf{G}$. In this case, we have that $\mathcal{U}(\mathbf{G}) = \emptyset$ and so the financial market is inefficient with respect to the information flow $\mathbf{G}$. Otherwise, the market participants would not be willing to pay any premium. This is a simple consequence of Theorem 6 after substituting $\mathbf{F}$ with $\mathbf{G}$.

Premiums are short-living in the sense of the following corollary.

Corollary 2. *Suppose that* $\mathcal{L}(\mathbf{G}) \neq \emptyset$ *and let the strategy of the fund manager be maximal with respect to* $\mathbf{G}$*. As soon as* $\{P_t\}$ *hits* $\{V_t\}$ *at any random time* $\varsigma \geq 0$*, we have that* $P_t = V_t$ *for all* $t \geq \varsigma$.

Proof: The strategy of the fund manager is maximal with respect to $\mathbf{G}$ and thus Theorem 9 guarantees that there exists some $\mathbb{Q} \in \mathcal{L}(\mathbf{G})$ such that the value process $\{V_t\}$ is a martingale with respect to $\mathbf{G}$. Further, $\{P_t\}$ is a positive local martingale and thus a supermartingale with respect to $\mathbf{G}$. This means that $\{P_t - V_t\}$ is a nonnegative supermartingale with respect to $\mathbf{G}$. It is well-known that each nonnegative supermartingale that hits zero must stay at zero. Put another way, we have that $P_t = V_t$ for all $t \geq \varsigma$. Q.E.D.

Hence, if the strategy of the manager is maximal with respect to $\mathbf{G}$ and the fund starts without a premium, we will neither observe a premium nor a discount in the course of time. Put another way, price and NAV must be congruent. Thus, if the market participants are willing to pay some premium for a maximal strategy, they must do this *from the outset*. This might explain why premiums can usually be observed during the first couple of months after the IPO, whereas they disappear later on and do no longer occur.

The following theorem does not require the manager's strategy to be maximal and can be considered a generalization of Corollary 2.

Theorem 10. *Suppose that* $\mathcal{L}(\mathbf{G}) \neq \emptyset$ *and fix any* $\mathbb{Q} \in \mathcal{L}(\mathbf{G})$*. Further, let the strategy of the fund manager be based on* $\mathbf{G}$.

- *Both the price process* $\{P_t\}$ *and the value process* $\{V_t\}$ *are bounded below by a uniformly integrable martingale* $\{L_t\}$ *with respect to* $\mathbf{G}$ *such that* $0 < L_0 \leq V_0 \leq P_0$ *and* $L_\tau = P_\tau = V_\tau$.
- *Further,* $\{L_t\}$ *absorbs* $\{P_t\}$ *and* $\{V_t\}$*. More precisely, it holds that*

  (i) $P_t = L_t$ *for all* $t \geq \varsigma$ *as soon as* $\{P_t\}$ *hits* $\{L_t\}$ *and*

(ii) $V_t = L_t$ *for all* $t \geq \varsigma$ *as soon as* $\{V_t\}$ *hits* $\{L_t\}$

at any random time $\varsigma \geq 0$.

Proof: The Ansel-Stricker theorem reveals that both the price process $\{P_t\}$ and the value process $\{V_t\}$ are positive supermartingales with respect to $\mathbf{G}$. Hence, from Doob's theorem, we conclude that P_∞ is integrable and

$$P_t \geq \mathbf{E}_{\mathbb{Q}}\big(P_\infty \,|\, \mathcal{G}_t\big) = \mathbf{E}_{\mathbb{Q}}\big(V_\infty \,|\, \mathcal{G}_t\big) \leq V_t \tag{14}$$

for all $t \geq 0$. This means that $\{P_t\}$ and $\{V_t\}$ are bounded below by the process $\{L_t\}$ with $L_t := \mathbf{E}_{\mathbb{Q}}\big(P_\infty \,|\, \mathcal{G}_t\big)$ for all $t \geq 0$, which is closed by P_∞ and thus a uniformly integrable martingale with respect to $\mathbf{G}$. From Theorem 1.42 in Jacod & Shiryaev (2003, Chapter I) it follows that $L_\tau = \mathbf{E}_{\mathbb{Q}}\big(L_\infty \,|\, \mathcal{G}_\tau\big)$ with $L_\infty = \mathbf{E}\big(P_\infty \,|\, \mathcal{G}\big) = P_\infty$ and thus

$$L_\tau = \mathbf{E}_{\mathbb{Q}}\big(P_\infty \,|\, \mathcal{G}_\tau\big) = \mathbf{E}_{\mathbb{Q}}\big(P_\tau \,|\, \mathcal{G}_\tau\big) = P_\tau = V_\tau. \tag{15}$$

Moreover, from Proposition 1 we know that $V_0 \leq P_0$. Thus, we obtain

$$L_0 = \mathbf{E}_{\mathbb{Q}}\big(P_\infty \,|\, \mathcal{G}_0\big) = \mathbf{E}_{\mathbb{Q}}\big(V_\infty \,|\, \mathcal{G}_0\big) \leq V_0 \leq P_0 \tag{16}$$

and, since $P_\infty > 0$, $L_0 = \mathbf{E}_{\mathbb{Q}}\big(P_\infty \,|\, \mathcal{G}_0\big) > 0$. Finally, since each nonnegative supermartingale that hits zero must stay at zero, $\{L_t\}$ absorbs $\{P_t\}$ and $\{V_t\}$. Q.E.D.

It is worth emphasizing that the lower bound that is mentioned by Theorem 10 depends on the choice of $\mathbb{Q}$ and thus, in general, it is not unique.

The theoretical results derived so far are depicted in Figure 2.

4.2.2. With Management Fee

In the last section, we investigated the case in which $\phi = 0$. Now, we assume that $\phi > 0$, i.e., that the manager receives a fee.

Theorem 11. *Let* $s > 0$ *be some future point in time before liquidation.*[20] *Suppose that* $\mathcal{L}(\boldsymbol{G}) \neq \emptyset$ *and fix any* $\mathbb{Q} \in \mathcal{L}(\boldsymbol{G})$. *Further, let the strategy of the fund manager be based on* $\boldsymbol{G}$ *and assume that* $\phi > 0$. *If* $\{P_t\}_{t \geq s}$ *is a uniformly integrable martingale with respect to* $\{\mathcal{G}_t\}_{t \geq s}$, *then* $P_t < W_t$ *for all* $s \leq t < \tau$.

Proof: If $\{P_t\}_{t \geq s}$ is a uniformly integrable martingale with respect to $\{\mathcal{G}_t\}_{t \geq s}$, we obtain

$$\begin{aligned} P_t &= \mathbf{E}_{\mathbb{Q}}\big(P_\tau \,|\, \mathcal{G}_t\big) = \mathbf{E}_{\mathbb{Q}}\big(W_\tau \,|\, \mathcal{G}_t\big) = \mathbf{E}_{\mathbb{Q}}\big(V_\tau e^{-\phi\tau} \,|\, \mathcal{G}_t\big) & (17) \\ &< \mathbf{E}_{\mathbb{Q}}\big(V_\tau e^{-\phi t} \,|\, \mathcal{G}_t\big) = \mathbf{E}_{\mathbb{Q}}\big(V_\tau \,|\, \mathcal{G}_t\big) e^{-\phi t} \leq V_t e^{-\phi t} = W_t & (18) \end{aligned}$$

for all $s \leq t < \tau$. Q.E.D.

[20] Hence, the statement of the theorem holds true conditional on $\{s < \tau\}$.

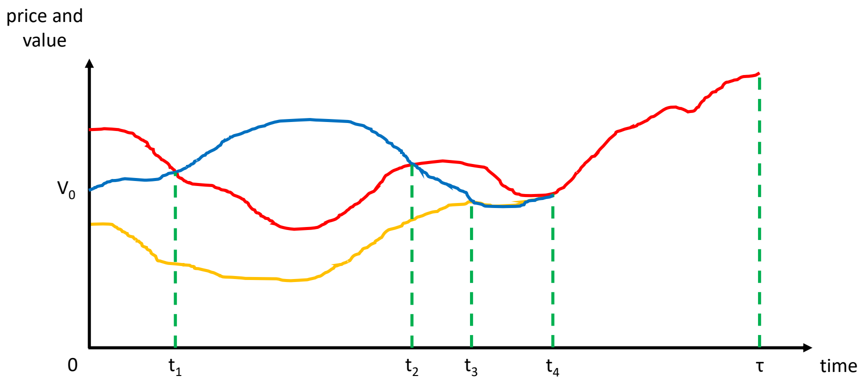

Figure 2: Lower bound (yellow), price (red), and value (blue) without management fee. The fund starts at a premium, which diminishes at time t_1. From time t_1 to t_2 we can observe a discount, which turns once again into a premium from time t_2 to t_4. From time t_4 to τ, i.e., the time of liquidation, price and value coincide. Now, from Part i of Theorem 9 we may conclude that the fund share cannot be maximal as long as we can observe a premium. Here, it cannot be maximal until time t_4. Further, Part ii of Theorem 9 implies that the strategy of the fund manager cannot be maximal before time t_2, i.e., when the discount turns into a premium. At time t_3 the value attains the lower bound, i.e., the strategy is maximal from time t_3 to τ. (Actually, Theorem 9 refers to the time point $t = 0$, but with a grain of salt it can be transferred, analogously, to the hitting time t_3. For this purpose, both price and value at time t_3 must be considered *fixed*.) However, at time t_3 the fund share is still overpriced with respect to **G**. Finally, at time t_4 also the fund share attains the lower bound and thus it is fairly priced from time t_4 to τ.

Theorem 11 strengthens our previous results: If the manager receives a fee, a discount *must* occur when the fund is fairly priced, and it cannot disappear until the time of liquidation.[21] Since we have that $P_0 \geq V_0 = W_0$, this situation can only appear *after* the IPO—otherwise the fund would have never been incepted. This is precisely the time at which the pricing formula proposed by Ross (2002, 2005), i.e.,

$$P_t = \frac{\gamma}{\gamma + \phi} \cdot W_t, \tag{19}$$

could be used, alternatively. However, this requires us to assume that the fund will never be liquidated. Instead, the investors receive a permanent dividend stream.

Obviously, Ross' pricing formula does not explain premiums,[22] and it tells us that the discount rate, i.e.,

$$\frac{W_t - P_t}{W_t} = \frac{\phi}{\gamma + \phi}, \tag{20}$$

is constant over time. Moreover, it is not a priori clear how much time after the IPO must elapse until we can apply Ross' pricing formula.

As long as we do not observe a discount, the fund share cannot be maximal and thus it is overpriced with respect to **G**. This implies that the financial market is inefficient with respect to **G**. Nonetheless, this does not mean that the market participants are irrational. A market participant who has access to the information flow **G** considers the fund share overpriced because he can apply the given strategy on his own. By contrast, another market participant who can exploit only of the information contained in **F** could be willing to accept the given situation and thus, from his own perspective, the fund share turns out to be fairly priced.

Theorem 12. *Suppose that $\mathcal{L}(\boldsymbol{G}) \neq \emptyset$ and fix any $\mathbb{Q} \in \mathcal{L}(\boldsymbol{G})$. Further, let the strategy of the fund manager be based on **G** and assume that $\phi > 0$.*

- *Both the price process $\{P_t\}$ and the wealth process $\{W_t\}$ are bounded below by a uniformly integrable martingale $\{L_t\}$ with respect to **G** such that $0 < L_0 < W_0 \leq P_0$ and $L_\tau = P_\tau = W_\tau$.*
- *Further, it holds that*
  - *(i) $P_t = L_t$ for all $t \geq \varsigma$ as soon as $\{P_t\}$ hits $\{L_t\}$ at any random time $\varsigma \geq 0$, whereas*
  - *(ii) $W_t > L_t$ for all $0 \leq t < \tau$ and $W_t = L_t$ for all $t \geq \tau$.*

Proof: Theorem 5 reveals that

$$L_t := \mathbf{E}_{\mathbb{Q}}\big(P_\infty \,|\, \mathcal{G}_t\big) = \mathbf{E}_{\mathbb{Q}}\big(W_\infty \,|\, \mathcal{G}_t\big) = \mathbf{E}_{\mathbb{Q}}\big(W_\tau \,|\, \mathcal{G}_t\big) < W_t \tag{21}$$

for all $0 \leq t < \tau$ and $W_t = L_t$ for all $t \geq \tau$, which proves Part ii of the second item. Part i of the second item follows from the proof of Theorem 10, and the same holds true for the first item after substituting $\{V_t\}$ with $\{W_t\}$. Q.E.D.

[21] If the price process of the fund starts to be a (uniformly integrable) martingale at some point in time, it cannot lose this property at any later time point.

[22] Nonetheless, Ross (2005) already observes that premiums may exist (only) if the manager applies an active trading strategy.

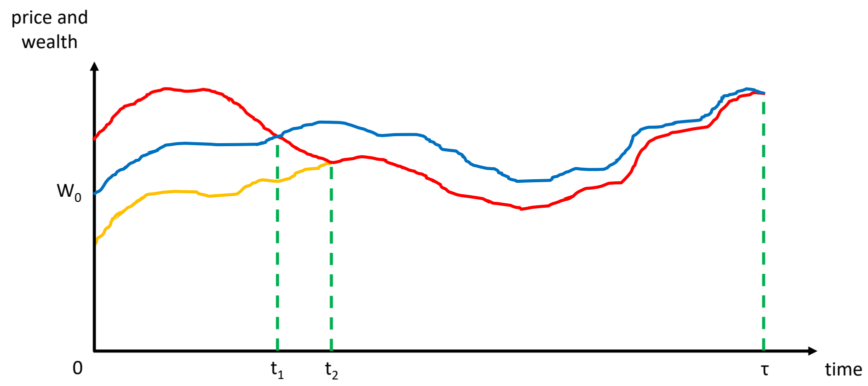

Figure 3: Lower bound (yellow), price (red), and wealth (blue) with management fee. The fund starts at a premium, which turns into a discount at time t_1. This lasts until time τ, i.e., the time of liquidation. At time t_2 the share price attains the lower bound. The fund share cannot be maximal until time t_1, whereas it must be maximal from t_2 to τ. By contrast, the wealth process always exceeds the lower bound before liquidation. Thus, we cannot say whether or not the strategy of the fund manager is maximal by visual inspection. Since the manager receives a fee, the wealth process cannot attain the lower bound before the fund is liquidated, even if his strategy *is* maximal.

Figure 3 illustrates typical paths of the lower bound, price, and wealth of a closed-end fund if the fund manager receives a fee.

Now, we would like to conclude this section by making some general remarks concerning the lifetime of premiums that can typically be observed at the beginning of a closed-end fund. The trading strategy of the fund manager is based on **G**. Hence, the Ansel-Stricker theorem tells us that the value process of his strategy is a positive local martingale and thus it is a supermartingale with respect to **G** for each $\mathbb{Q} \in \mathcal{L}(\mathbf{G})$. This means that it is still a supermartingale with respect to $\{\mathcal{G}_t\}_{t\geq s}$ for every future point in time $s > 0$. Moreover, if the strategy of the fund manager is based only on $\{\mathcal{F}_t\}_{t\geq s}$ from time s to τ, then its value process $\{V_t\}_{t\geq s}$ is a supermartingale with respect to $\{\mathcal{F}_t\}_{t\geq s}$.[23] This leads us to two basic hypotheses why premiums are short-living in real-life financial markets:

A. Suppose that $\mathbb{Q} \in \mathcal{U}(\mathbf{F})$. Hence, there is NWA based on $\{\mathcal{F}_t\}_{t\geq s}$, i.e., $\{P_t\}_{t\geq s}$ is a uniformly integrable martingale with respect to $\{\mathcal{F}_t\}_{t\geq s}$. Further, let the strategy of the fund manager be based on $\{\mathcal{F}_t\}_{t\geq s}$ from time s to τ. Hence, during that period, he applies a passive strategy, whose value process $\{V_t\}_{t\geq s}$ is a supermartingale with respect to $\{\mathcal{F}_t\}_{t\geq s}$. Since the fund manager receives a fee, i.e., $\phi > 0$, we obtain

$$\begin{aligned} W_s &= V_s e^{-\phi s} \geq \mathbf{E}_{\mathbb{Q}}\big(V_\tau e^{-\phi s} \,|\, \mathcal{F}_s\big) && (22) \\ &> \mathbf{E}_{\mathbb{Q}}\big(V_\tau e^{-\phi \tau} \,|\, \mathcal{F}_s\big) = \mathbf{E}_{\mathbb{Q}}\big(W_\tau \,|\, \mathcal{F}_s\big) = \mathbf{E}_{\mathbb{Q}}\big(P_\tau \,|\, \mathcal{F}_s\big) = P_s. && (23) \end{aligned}$$

We conclude that the fund must trade at a discount.

B. Now, suppose that there is NWA based on $\{\mathcal{G}_t\}_{t\geq s}$, i.e., $\{P_t\}_{t\geq s}$ is a uniformly integrable

[23]Remember that $\mathcal{L}(\mathbf{G}) \subseteq \mathcal{L}(\mathbf{F})$, i.e., $\mathbb{Q}$ is a local martingale measure with respect to **F**, too.

martingale and the value process $\{V_t\}_{t\geq s}$ is a supermartingale with respect to $\{\mathcal{G}_t\}_{t\geq s}$ under some $\mathbb{Q}\in\mathcal{U}(\{\mathcal{G}_t\}_{t\geq s})$.[24] Thus, we have that

$$\begin{aligned} P_s &= \mathbf{E}_{\mathbb{Q}}(P_\tau \mid \mathcal{G}_s) = \mathbf{E}_{\mathbb{Q}}(W_\tau \mid \mathcal{G}_s) = \mathbf{E}_{\mathbb{Q}}(V_\tau e^{-\phi\tau} \mid \mathcal{G}_s) && (24)\\ &< \mathbf{E}_{\mathbb{Q}}(V_\tau e^{-\phi s} \mid \mathcal{G}_s) \leq V_s e^{-\phi s} = W_s. && (25) \end{aligned}$$

The result is the same as in Hypothesis **A**, i.e., the fund must trade at a discount.

The given hypotheses are similar from a technical viewpoint, but they differ essentially from an economic perspective. This will be discussed in more detail in the following section.

5. A Practical Example

We would like to demonstrate our theoretical findings by means of a simple example. This example shall serve only as a basic model in order to illustrate the CEFP and to clarify some questions that could still be open. In principle, the given example, or any modification of it, might be used also for an empirical investigation of the CEFP. Here, our main goal is to show that our theoretical findings can be used in order to explain the magnitude of discounts and premiums that is typically observed in real-life financial markets. However, in order to accomplish this goal, one has to make some additional assumptions, which is done in this section.

Suppose that the financial market has a finite lifetime $T > 0$. Let $\mathbf{F}$ be the flow of public information, i.e., the information that is available to everyone. Consider some person with private information flow $\mathbf{G}\supset\mathbf{F}$ who is able to produce abnormal profits by applying a trading strategy based on $\mathbf{G}$. To explain the meaning of "abnormal profits," let us fix some risk-neutral measure $\mathbb{Q}\in\mathcal{U}(\mathbf{F})$, which implies that there is NWA with respect to $\mathbf{F}$. Let

$$\frac{dS_{it}}{S_{it}} = \sigma_i\, dB_{it} \tag{26}$$

be the instantaneous rate of return on asset $i = 1, 2, \ldots, N$, where $\{B_{it}\}$ represents a standard Brownian motion under the risk-neutral measure $\mathbb{Q}$ and with respect to the filtration $\mathbf{F}$. Further, $\sigma_i > 0$ denotes the volatility of the risky asset i. Hence, its price at time t is

$$S_{it} = S_{i0}\exp\left(-\frac{1}{2}\sigma_i^2 t + \sigma_i B_{it}\right) \tag{27}$$

and $\{S_{it}\}$ is a uniformly integrable martingale with respect to $\mathbf{F}$.[25]

We pointed out that the person mentioned above is able to perform a superior strategy, which is based on his private information flow $\mathbf{G}$. Let $\{V_t\}$ be the value process of that strategy. More

[24] To be more precise, we have to treat the future time point s like time 0, i.e., we must consider all asset prices, the value of the strategy, and thus also the wealth of the fund at time s *fixed*.

[25] Note that we assume that the financial market has a finite lifetime.

precisely, let

$$\frac{dV_t}{V_t} = \mu_t \, dt + \sigma_t \, dB_t \tag{28}$$

be its instantaneous rate of return, where both the drift process $\{\mu_t\}$ and the (nonnegative) diffusion process $\{\sigma_t\}$ are **G**-predictable. Moreover, $\{B_t\}$ is a standard Brownian motion with respect to **F**. How can we interpret the given assumptions?

The considered person permanently decides to buy and sell some assets by using *any* kind of information that is contained in **G**. Thus, he can freely decide which part of his private information to exploit. His portfolio at each time t depends only on his private information at any time *before* t. This means that the trading strategy is **G**-predictable—not only **G**-adapted. Depending on the long and short positions that the person holds at time t, he implicitly generates some drift $\mu_t \in \mathbb{R}$ and some diffusion $\sigma_t \geq 0$. It is clear that the drift and diffusion coefficients depend only on his private information before t. That is, whenever we know his private information that is available before time t, we know his portfolio at time t and thus $\{(\mu_t, \sigma_t)\}$ is **G**-predictable. The financial market is always in equilibrium, i.e., it clears at every time $t \geq 0$, *including* the transactions of the person who is taken into consideration.

Recall that the person is able to achieve abnormal profits. Our arguments are based on a risk-neutral measure and they refer to the *discounted* price process $\{S_t\}$. Hence, achieving abnormal profits just means to generate a positive drift μ_t during any time interval between time 0 and T. How can this be possible, given that there is NWA with respect to the public information flow **F**? The person addressed exploits information that goes *beyond* **F** and thus we could assume that he can find a free lunch with vanishing risk on the basis of his private information flow **G**. However, this assumption might be too optimistic in our neoclassical framework, in which NFLVR represents a minimum requirement. Alternatively, we could assume that he can create a dominant strategy, i.e., explore mispriced assets, on the basis of **G**. It is worth emphasizing that those assets are mispriced only with respect to **G** but not with respect to **F**. This means that the market could be efficient with respect to the public information flow **F** but inefficient with respect to the private information flow **G**. Thus, achieving abnormal profits could clearly be possible for everybody who has access to the information contained in **G**.

Our person would like to make capital out of his special capabilities and so he decides to act as a fund manager. Of course, instead he could apply the same strategy for himself. We do not presume that his special capabilities just appear when managing a fund. That is, in general, he is able to produce abnormal profits even *before* he starts working as a fund manager. Nonetheless, it would not harm at all to assume that our person gets access to substantial information just *because* he is working in the company as a fund manager. In any case, receiving a management fee *and* investing some money in the own fund might be better for him than receiving no fee and trading the strategy on his own. This depends very much on the trading volume, the profitability of his strategy, and the management fee. This is a typical micro-economic decision problem. Here, we assume that it is actually better for the person to act as a fund manager.

The IPO takes place at time $t = 0$ and the management fee is $\phi > 0$. Let us assume that the fund is going to be liquidated at time T. From Itô's lemma it follows that

$$V_t = V_0 \exp\left(\int_0^t \left(\mu_s - \frac{1}{2}\sigma_s^2\right) ds + \int_0^t \sigma_s \, dB_s\right) \tag{29}$$

for all $0 \leq t \leq T$. Hence, the wealth of the fund at time t is

$$\begin{aligned} W_t &= V_0 \exp\left(\int_0^t \left(\mu_s - \frac{1}{2}\sigma_s^2\right) ds + \int_0^t \sigma_s \, dB_s\right) e^{-\phi t} & (30) \\ &= V_0 \exp\left(\int_0^t \left(\mu_s - \phi - \frac{1}{2}\sigma_s^2\right) ds + \int_0^t \sigma_s \, dB_s\right). & (31) \end{aligned}$$

The fund manager exploits of his private information flow $\mathbf{G}$. Nonetheless, we could ask whether he *must* use any kind of information that goes beyond the public information flow $\mathbf{F} \subset \mathbf{G}$. We already know from Theorem 7 that he cannot receive any fee if he applies a passive strategy, i.e., a strategy that is based on $\mathbf{F}$, and so the answer is "Yes"! To be more precise, the fund would have never been incepted if the manager had planned to trade only on the basis of public information. This does not mean that he must make use of nonpublic information all the time. It might very well suffice to perform an active strategy, i.e., a strategy that makes use of information that goes beyond $\mathbf{F}$, anytime between $t = 0$ and $t = T$.

This fundamental result can be interpreted in terms of μ_t. The fund would not exist in the first place if $\{\mu_t\}$ were a nondegenerate stochastic process on $[0, T)$ that is bounded above by ϕ. It is easy to see that in this case we would have that

$$P_0 = \mathbf{E}_{\mathbb{Q}}(P_T \mid \mathcal{F}_0) = \mathbf{E}_{\mathbb{Q}}(W_T \mid \mathcal{F}_0) < V_0 \,, \tag{32}$$

but due to Proposition 1 it is impossible that the fund starts with a discount. Of course, also in the degenerate case $\mu_t \equiv \phi$, there is no economic reason for an IPO. We conclude that it must hold that $\mu_t > \phi$, at least temporarily, since otherwise nobody would be willing to pay the management fee. Hence, at least for a short period of time, the fund manager must be able to produce abnormal profits in order to compensate for his management fee and thus to guarantee that anybody is willing to invest in the fund right from the start.

Let us assume that $\sigma_t \equiv \sigma > 0$ for the sake of simplicity. Suppose that $\{\mu_t\}$ starts with some initial drift $\mu_0 > 0$. Then the drift decreases by and by until it approaches zero at some point in time $s < T$, i.e., before the fund is liquidated. Hence, the performance of the fund manager diminishes over time until it reaches the fair market level. Why does this happen? We have already provided two possible answers to this question in the previous section: Hypothesis **A** and Hypothesis **B**.

The former hypothesis states that the fund manager starts to ignore the surplus information that is contained in $\mathbf{G}$ at time s (or even before) in order to rely upon public information only. This decision could be rational from the perspective of the fund manager if searching for valuable information is costly. Hence, it might be better to stop searching for extraordinary information and to apply a passive strategy, which is based only on public information by its very definition. Note that the decision when to stop searching for extraordinary information depends on the manager's *own* utility function and his choice can have an essential impact on the aggregate demand function regarding the fund share (see Section 3.1). The crucial point is that, in any case, the fund manager receives his management fee until the time of liquidation—irrespective of whether he performs well or not. This means that he is facing a trade-off between making an effort and losing his reputation. In the end, his personal decision can be explained by micro-economic arguments. More precisely, this is a principal-agent problem with hidden action (Holmström, 1979) because the investors might know the wealth process of the fund, but in general they do not know the strategy of the fund manager. In any case, we need not assume that μ_t suddenly drops down to zero at time s.

By contrast, the latter hypothesis states that the fund manager keeps going on using surplus information, but at some point in time after the IPO, he is no longer able to find any mispriced assets on the basis of **G**. How can this happen? First of all, we could imagine that his private information turns into public information as time goes by. Then, however, we would have to give the filtration **F** another name, but this is only a terminological problem. Indeed, it seems not unusual that any kind of information becomes widespread in our digitalized world. For example, we could think about some insider who leaks some essential details of the manager's strategy. The more subjects know that strategy, or the higher their trading volumes, the more it is likely that demand and supply lead to a situation in which the strategy is no longer profitable from time s (or even before) to time T. Another possibility is that somebody else than the fund manager learns the same strategy just by himself. People try to find profitable strategies since financial markets exist and so it is not surprising if they come to the same idea a bit at a time. According to Lo's (2004) adaptive-market hypothesis, a financial market represents an evolutionary system, i.e., forces like competition, reproduction, and natural selection are at work. Hence, it is subject to a permanent change and progress, which means that abnormal profits can very well exist, but they are transient in a competitive financial market.

Hypothesis **A** uses a principal-agent argument, whereas Hypothesis **B** focuses on market efficiency or, at least, on the absence of weak arbitrage opportunities. It is worth emphasizing that our hypotheses do not contradict one another and thus can be coexistent. On the one hand, we can very well imagine that the fund manager becomes "lazy," simply because doing his best until the time of liquidation is suboptimal. On the other hand, since financial markets are very competitive, it seems to be plausible also that a fund manager can achieve abnormal profits only for a short period of time, i.e., as long as other market participants are unable to exploit the corresponding information. The question of whether the first or the second hypothesis is more appropriate can only be investigated empirically and goes beyond the scope of this work.

A simple model for the drift process $\{\mu_t\}$ is $\mu_t = \max\{\mu_0(1 - t/s), 0\}$, which implies that the drift coefficient reaches the fair market level at time s. Here, we choose $s = T/2$, without loss of generality, and so we obtain

$$W_t = V_0 \exp\left(\mu_0 \left(1 - \frac{t \wedge \frac{T}{2}}{T}\right)\left(t \wedge \frac{T}{2}\right) - \phi t - \frac{1}{2}\sigma^2 t + \sigma B_t\right) \tag{33}$$

with $t \wedge \frac{T}{2} = \min\{t, T/2\}$. Thus, we have that

$$W_T = V_0 \exp\left(\left(\frac{\mu_0}{4} - \phi\right) T - \frac{1}{2}\sigma^2 T + \sigma B_T\right), \tag{34}$$

which leads us to the initial price

$$P_0 = \mathbf{E}_{\mathbb{Q}}\big(P_T \,|\, \mathcal{F}_0\big) = \mathbf{E}_{\mathbb{Q}}\big(W_T \,|\, \mathcal{F}_0\big) = V_0 \exp\left(\left(\frac{\mu_0}{4} - \phi\right) T\right). \tag{35}$$

This means that the fund manager must start with some drift coefficient μ_0 that is at least four times greater than his management fee because otherwise $P_0 < V_0$.

Note that

$$\begin{aligned} P_t &= \mathbf{E}_{\mathbb{Q}}\big(P_T \,|\, \mathcal{F}_t\big) = \mathbf{E}_{\mathbb{Q}}\big(W_T \,|\, \mathcal{F}_t\big) && (36)\\ &= V_0 \exp\left(\left(\frac{\mu_0}{4} - \phi\right) T - \frac{1}{2}\sigma^2 t + \sigma B_t\right) && (37)\\ &= W_t \exp\left(\mu_0 \left(\frac{T}{4} - \left(1 - \frac{t \wedge \frac{T}{2}}{T}\right)\left(t \wedge \frac{T}{2}\right)\right) - \phi(T - t)\right) && (38) \end{aligned}$$

and consider the discount rate

$$\frac{W_t - P_t}{W_t} = 1 - \exp\left(\mu_0 \left(\frac{T}{4} - \left(1 - \frac{t \wedge \frac{T}{2}}{T}\right)\left(t \wedge \frac{T}{2}\right)\right) - \phi(T - t)\right), \tag{39}$$

which takes on a negative value when the fund trades at a premium and a positive one when it trades at a discount.

Note that the turning time depends only on T, ϕ, and μ_0 because we assume that the drift process $\{\mu_t\}$ is deterministic and the volatility process $\{\sigma_t\}$ is constant. This model enables us to calculate also the time at which the discount rate reaches its maximum, i.e.,

$$t^* = \left(\frac{\mu_0 - \phi}{\mu_0}\right)\frac{T}{2}, \tag{40}$$

which clearly reveals that this happens before the fund manager stops performing a superior strategy. It is precisely that time at which μ_t equals ϕ, i.e., when the drift coefficient starts to fall below the management fee.

Figure 4 depicts the evolution of the discount rate for different values of ϕ and μ_0 by assuming that the fund is liquidated after 20 years. As we can see, the discounts and premiums are in accordance with most empirical observations. We could have used any other deterministic or stochastic model for $\{(\mu_t, \sigma_t)\}$. An extension of our simple setup might involve modeling μ_t as a function of exogenous variables. For example, in order to shed some light on whether Hypothesis **A** or Hypothesis **B** is more appropriate, we could assume that the drift coefficient depends on the manager's search costs and the innovation level of his strategy.[26] Although that setup is more sophisticated, we think that the overall conclusion, namely that discounts and

[26] Admittedly, adequate measures for those variables might be hard to obtain in practice.

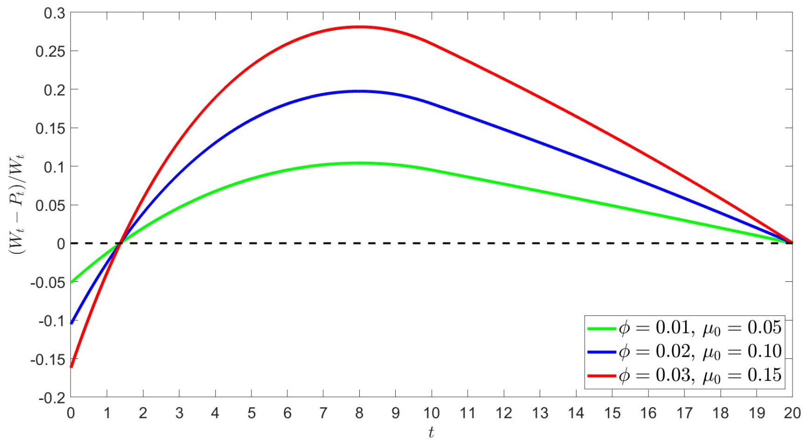

Figure 4: Discount rate of the fund over its lifetime. It trades at a premium during the first 342 trading days. Hence, almost one and a half year after its IPO, the premium turns into a discount. The largest discount occurs after 8 years. Afterwards, it tends slowly to zero as the fund approaches its time of liquidation. For example, in the moderate case $\phi = 0.02$ and $\mu_0 = 0.10$, the fund starts with a discount rate of almost –11% and the largest discount rate is approximately 20%.

premiums can be explained by no-arbitrage theory, would remain unchanged.

6. Conclusion

The existence of discounts and premiums does not violate fundamental principles of neoclassical finance, i.e., no arbitrage and rationality. We explain the main characteristics of the CEFP by applying fundamental theorems of asset pricing. The 1st FTAP enables us to clarify why premiums can occur in a financial market without free lunches with vanishing risk. By contrast, the 3rd FTAP reveals why discounts are more prevalent than premiums. Our results are consistent with both the time-series and the cross-sectional aspect of the CEFP.

A major observation of our theory is that a closed-end fund can never start at a discount. Moreover, if the fund manager receives a fee, the fund can only exist if he has access to information that enables him to create a weak arbitrage, i.e., a free lunch with vanishing risk or a dominant strategy. The former can be considered a possibility to make money out of nothing while keeping the maximum loss arbitrarily small, whereas the latter implies that at least one asset in the market is mispriced. Hence, a closed-end fund can only exist if the fund manager performs an active trading strategy, at least for a short period of time.

The premium at the IPO of a closed-end fund represents a remuneration of the founders for finding a manager who is willing and able to exploit extraordinary information. However, a premium can no longer occur as soon as the arbitrage opportunities of the manager evaporate, i.e., after the price of the fund attains a lower bound, which indicates that the market participants are no longer ecstatic about its future prospects. The reason why a premium quickly turns into a discount might be that the fund manager stops applying a superior trading strategy at some point in time or that abnormal profits are, in general, transient in a competitive financial market.

When the manager is no longer willing or able to maintain a superior strategy, the fund must trade at a discount in order to compensate for his management fee.

References

A. Abraham, D. Elan & A. Marcus (1993) Does sentiment explain closed-end fund discounts? Evidence from bond funds, *Financial Review* **28**, 607–616.

J.-P. Ansel & C. Stricker (1994) Couverture des actifs contingents et prix maximum, *Annales de l'Institut Henri Poincaré* **30**, 303–315.

N. Barberis, A. Shleifer & R. Vishny (1998) A model of investor sentiment, *Journal of Financial Economics* **49**, 307–343.

M.J. Barclay, C.G. Holderness & J. Pontiff (1993) Private benefits from block ownership and discounts on closed-end funds, *Journal of Financial Economics* **33**, 263–291.

J.B. Berk & R. Stanton (2007) Managerial ability, compensation, and the closed-end fund discount, *Journal of Finance* **62**, 529–556.

J. Bodurtha, D.S. Kim & C. Lee (1995) Closed-end country funds and U.S. market sentiment, *Review of Financial Studies* **8**, 879–918.

J. Brickley, S. Manaster & J. Schallheim (1991) The tax-timing option and the discounts on closed-end investment companies, *Journal of Business* **64**, 287–312.

J. Chan, R. Jain & J. Xia (2008) Market segmentation, liquidity spillover, and closed-end country fund discounts, *Journal of Financial Markets* **11**, 377–399.

E. Chang, C.S. Eun & R. Kolodny (1995) International diversification through closed-end country funds, *Journal of Banking and Finance* **19**, 1237–1263.

M. Charrón (2009) The closed-end funds puzzle: a survey review, *Forum Empresarial* **14**, 39–57.

J.B. Chay & C.A. Trzcinka (1999) Managerial performance and the cross-sectional pricing of closed-end funds, *Journal of Financial Economics* **52**, 379–408.

M. Cherkes (2012) Closed-end funds: a survey, *Annual Review of Financial Economics* **4**, 431–445.

M. Cherkes, J. Sagi & R. Stanton (2009) A liquidity-based theory of closed-end funds, *Review of Financial Studies* **22**, 257–297.

V. Datar (2001) Impact of liquidity on premia/discounts in closed-end funds, *Quarterly Review of Economics and Finance* **41**, 119–135.

J.B. de Long, A. Shleifer, L.H. Summers & R.J. Waldmann (1990) Noise trader risk in financial markets, *Journal of Political Economy* **98**, 703–738.

F. Delbaen & W. Schachermayer (1994) A general version of the fundamental theorem of asset pricing, *Mathematische Annalen* **300**, 463–520.

F. Delbaen & W. Schachermayer (1997a) The Banach space of workable contingent claims in arbitrage theory, *Annales de l'Institut Henri Poincaré* **1**, 114–144.

F. Delbaen & W. Schachermayer (1997b) Non-arbitrage and the fundamental theorem of asset pricing: summary of main results. In: *Introduction to Mathematical Finance* (D.C. Heath & G. Swindle, eds.), 49–59. American Mathematical Society.

F. Delbaen & W. Schachermayer (1998) The fundamental theorem of asset pricing for unbounded stochastic processes, *Mathematische Annalen* **312**, 215–250.

F. Delbaen & W. Schachermayer (2001) Applications to Mathematical Finance. In: *Handbook of the Geometry of Banach Spaces* (W.B. Johnson & J. Lindenstrauss, eds.), 367–391. Elsevier.

E. Dimson & C. Minio-Kozerski (1999) Closed-end funds: a survey, *Financial Markets, Institutions and Instruments* **8**, 1–41.

E. Elton, M. Gruber & J. Busse (1998) Do investors care about sentiment?, *Journal of Business* **71**, 477–500.

R. Ferguson & D. Leistikow (2001) Valuing active managers, fees, and fund discounts, *Financial Analysts Journal* **57**, 52–62.

G. Frahm (2016) Pricing and valuation under the real-world measure, *International Journal of Theoretical and Applied Finance* **19**. doi: 10.1142/S0219024916500060.

G. Frahm (2018) Corrigendum: 'Pricing and valuation under the real-world measure', *International Journal of Theoretical and Applied Finance* **21**. doi: 10.1142/S0219024918920012.

U. Garay & P. Russel (1999) The closed-end fund puzzle: a review, *Journal of Alternative Investments* **2**, 23–44.

G. Gemmill & D.C. Thomas (2002) Noise trading, costly arbitrage, and asset prices: evidence from closed-end funds, *Journal of Finance* **57**, 2571–2594.

K.W. Hanley, C.M.C. Lee & P.J. Seguin (1994) The marketing of closed-end fund IPOs: evidence from transactions data, The Wharton School, University of Pennsylvania.

J.M. Harrison & D.M. Kreps (1979) Martingales and arbitrage in multiperiod securities markets, *Journal of Economic Theory* **20**, 381–408.

J.M. Harrison & S.R. Pliska (1981) Martingales and stochastic integrals in the theory of continuous trading, *Stochastic Processes and their Applications* **11**, 215–260.

J.M. Harrison & S.R. Pliska (1983) A stochastic calculus model of continuous trading: complete markets, *Stochastic Processes and their Applications* **15**, 313–316.

B. Holmström (1979) Moral hazard and observability, *Bell Journal of Economics* **10**, 74–91.

J. Jacod (1979) *Calcul Stochastique et Problèmes de Martingales*. Lecture Notes in Mathematics **714**. Springer.

J. Jacod & A.N. Shiryaev (2003) *Limit Theorems for Stochastic Processes*, second edition. Springer.

R. Jarrow (2012) The third fundamental theorem of asset pricing, *Annals of Financial Economics* **7**. doi: 10.1142/S2010495212500078.

R. Jarrow & M. Larsson (2012) The meaning of market efficiency, *Mathematical Finance* **22**, 1–30.

R. Jarrow & P. Protter (2019) A rational asset pricing model for premiums and discounts on closed-end funds: the bubble theory, *Mathematical Finance*. doi: 10.1111/mafi.12207.

C. Lee, A. Shleifer & R.H. Thaler (1991) Investor sentiment and the closed-end fund puzzle, *Journal of Finance* **46**, 75–109.

A.W. Lo (2004) The adaptive markets hypothesis, *Journal of Portfolio Management* **30**, 15–29.

B. Malkiel (1977) The valuation of closed-end investment-company shares, *Journal of Finance* **32**, 847–859.

B. Malkiel (1995) The structure of closed end fund discounts revisited, *Journal of Portfolio Management* **21**, 32–38.

R.C. Merton (1973) Theory of rational option pricing, *Bell Journal of Economics and Management Science* **4**, 141–183.

J. Pontiff (1996) Costly arbitrage: evidence from closed-end funds, *Quarterly Journal of Economics* **111**, 1135–1151.

T. Ramadorai (2012) The secondary market for hedge funds and the closed hedge fund premium, *Journal of Finance* **67**, 479–512.

R.L. Roenfeldt & D.L. Tuttle (1973) An examination of the discounts and premiums of closed-end investment companies, *Journal of Business Research* **1**, 129–140.

S.A. Ross (2002) Neoclassical finance, alternative finance, and the closed-end fund puzzle, *European Financial Management* **8**, 129–137.

S.A. Ross (2005) *Neoclassical Finance*. Princeton University Press.