# Supplementary Material:
# Analysis and Forecasting of Risk in Count Processes

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## Contents



### Acronyms

DGP = data-generating process
Poi = Poisson
ZIP = zero-inflated Poisson
NB = negative binomial
INAR = integer-valued autoregressive
INARCH = integer-valued autoregressive conditional heteroscedasticity
BinAR = binomial autoregressive
BinARCH = binomial autoregressive conditional heteroscedasticity
ll-Poi-AR = log-linear Poisson autoregressive

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## S.1 Figures with Boxplots for Section 4

See the content of the zip-file "RiskPredCountTS_Boxplots.zip".

## S.2 Tables with Summarizing Statistics for Section 4

### Section 4.1 INAR(1) Count DGPs

Table S.1: Relative frequency of risk underrating for Poi-, ZIP, and NB-INAR(1) DGPs, coherent ("Coh") vs. approximate ("Gau") forecasts, for different $(\alpha, T)$, $I = 1.4$ in case of overdispersion and forecast horizon $h = 1$, computed across all simulation runs and all levels of $\mu$.

| | Poi-INAR(1) DGP | | | | | | ZIP-INAR(1) DGP | | | | | | NB-INAR(1) DGP | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\alpha = 0.33$ | | $\alpha = 0.55$ | | $\alpha = 0.80$ | | $\alpha = 0.33$ | | $\alpha = 0.55$ | | $\alpha = 0.80$ | | $\alpha = 0.33$ | | $\alpha = 0.55$ | | $\alpha = 0.80$ | |
| | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau |
| $\mathbf{VaR_{0.95}}$ | | | | | | | | | | | | | | | | | | |
| $T = 75$ | 0.171 | 0.631 | 0.142 | 0.587 | 0.076 | 0.485 | 0.203 | 0.559 | 0.179 | 0.586 | 0.154 | 0.639 | 0.261 | 0.704 | 0.231 | 0.666 | 0.170 | 0.587 |
| $T = 250$ | 0.099 | 0.663 | 0.086 | 0.608 | 0.061 | 0.498 | 0.120 | 0.582 | 0.111 | 0.626 | 0.115 | 0.722 | 0.170 | 0.789 | 0.160 | 0.753 | 0.125 | 0.676 |
| $T = 2500$ | 0.034 | 0.685 | 0.030 | 0.619 | 0.026 | 0.505 | 0.038 | 0.586 | 0.035 | 0.643 | 0.040 | 0.760 | 0.053 | 0.857 | 0.049 | 0.809 | 0.042 | 0.718 |
| $\mathbf{TCE_{0.95}}$ | | | | | | | | | | | | | | | | | | |
| $T = 75$ | 0.475 | 0.527 | 0.408 | 0.458 | 0.284 | 0.328 | 0.486 | 0.426 | 0.451 | 0.454 | 0.447 | 0.538 | 0.529 | 0.731 | 0.522 | 0.703 | 0.500 | 0.647 |
| $T = 250$ | 0.482 | 0.525 | 0.443 | 0.435 | 0.373 | 0.299 | 0.498 | 0.400 | 0.480 | 0.453 | 0.521 | 0.610 | 0.532 | 0.832 | 0.542 | 0.809 | 0.560 | 0.770 |
| $T = 2500$ | 0.495 | 0.526 | 0.480 | 0.418 | 0.461 | 0.281 | 0.500 | 0.361 | 0.494 | 0.434 | 0.519 | 0.644 | 0.510 | 0.924 | 0.515 | 0.892 | 0.534 | 0.842 |
| $\mathbf{ES_{0.95}}$ | | | | | | | | | | | | | | | | | | |
| $T = 75$ | 0.495 | 0.758 | 0.440 | 0.697 | 0.308 | 0.603 | 0.499 | 0.635 | 0.474 | 0.667 | 0.442 | 0.760 | 0.522 | 0.881 | 0.510 | 0.871 | 0.478 | 0.864 |
| $T = 250$ | 0.497 | 0.861 | 0.464 | 0.779 | 0.388 | 0.682 | 0.506 | 0.697 | 0.496 | 0.747 | 0.509 | 0.894 | 0.526 | 0.976 | 0.535 | 0.974 | 0.546 | 0.981 |
| $T = 2500$ | 0.499 | 0.964 | 0.489 | 0.863 | 0.466 | 0.763 | 0.503 | 0.742 | 0.500 | 0.807 | 0.513 | 0.971 | 0.508 | 1.000 | 0.512 | 1.000 | 0.528 | 1.000 |
| $\mathbf{MVaR_{0.95}}$ | | | | | | | | | | | | | | | | | | |
| $T = 75$ | 0.500 | 0.752 | 0.456 | 0.727 | 0.333 | 0.719 | 0.502 | 0.653 | 0.484 | 0.694 | 0.448 | 0.770 | 0.518 | 0.778 | 0.504 | 0.757 | 0.467 | 0.723 |
| $T = 250$ | 0.499 | 0.854 | 0.474 | 0.820 | 0.404 | 0.829 | 0.507 | 0.724 | 0.500 | 0.787 | 0.506 | 0.911 | 0.522 | 0.905 | 0.526 | 0.892 | 0.523 | 0.886 |
| $T = 2500$ | 0.500 | 0.962 | 0.492 | 0.910 | 0.470 | 0.928 | 0.503 | 0.789 | 0.502 | 0.863 | 0.510 | 0.987 | 0.506 | 0.996 | 0.510 | 0.991 | 0.516 | 0.995 |
| $\mathbf{EVaR_{0.95}}$ | | | | | | | | | | | | | | | | | | |
| $T = 75$ | 0.501 | 0.661 | 0.467 | 0.626 | 0.362 | 0.556 | 0.503 | 0.587 | 0.488 | 0.618 | 0.452 | 0.683 | 0.513 | 0.739 | 0.501 | 0.724 | 0.467 | 0.707 |
| $T = 250$ | 0.500 | 0.748 | 0.480 | 0.690 | 0.420 | 0.610 | 0.504 | 0.641 | 0.499 | 0.693 | 0.502 | 0.822 | 0.519 | 0.868 | 0.523 | 0.861 | 0.525 | 0.868 |
| $T = 2500$ | 0.500 | 0.891 | 0.494 | 0.788 | 0.477 | 0.682 | 0.503 | 0.698 | 0.501 | 0.780 | 0.509 | 0.945 | 0.505 | 0.993 | 0.508 | 0.987 | 0.518 | 0.994 |

Table S.2: Mean severity of risk underrating for Poi-, ZIP, and NB-INAR(1) DGPs, coherent ("Coh") vs. approximate ("Gau") forecasts, for different $(\alpha, T)$, $I = 1.4$ in case of overdispersion and forecast horizon $h = 1$, computed across all simulation runs and all levels of $\mu$.

| | Poi-INAR(1) DGP | | | | | | ZIP-INAR(1) DGP | | | | | | NB-INAR(1) DGP | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\alpha = 0.33$ | | $\alpha = 0.55$ | | $\alpha = 0.80$ | | $\alpha = 0.33$ | | $\alpha = 0.55$ | | $\alpha = 0.80$ | | $\alpha = 0.33$ | | $\alpha = 0.55$ | | $\alpha = 0.80$ | |
| | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau |
| $\mathbf{VaR_{0.95}}$ | | | | | | | | | | | | | | | | | | |
| $T = 75$ | -1.034 | -0.582 | -1.020 | -0.538 | -1.005 | -0.424 | -1.049 | -0.570 | -1.026 | -0.579 | -1.018 | -0.530 | -1.110 | -0.742 | -1.068 | -0.682 | -1.017 | -0.529 |
| $T = 250$ | -1.002 | -0.444 | -1.001 | -0.416 | -1.000 | -0.332 | -1.005 | -0.428 | -1.002 | -0.455 | -1.001 | -0.452 | -1.014 | -0.586 | -1.006 | -0.553 | -1.000 | -0.446 |
| $T = 2500$ | -1.000 | -0.378 | -1.000 | -0.357 | -1.000 | -0.292 | -1.000 | -0.354 | -1.000 | -0.387 | -1.000 | -0.410 | -1.000 | -0.489 | -1.000 | -0.463 | -1.000 | -0.392 |
| $\mathbf{TCE_{0.95}}$ | | | | | | | | | | | | | | | | | | |
| $T = 75$ | -0.402 | -0.553 | -0.386 | -0.499 | -0.304 | -0.371 | -0.466 | -0.525 | -0.439 | -0.534 | -0.407 | -0.503 | -0.710 | -0.842 | -0.649 | -0.779 | -0.531 | -0.627 |
| $T = 250$ | -0.224 | -0.384 | -0.219 | -0.352 | -0.190 | -0.256 | -0.260 | -0.368 | -0.254 | -0.395 | -0.275 | -0.404 | -0.437 | -0.667 | -0.433 | -0.636 | -0.402 | -0.536 |
| $T = 2500$ | -0.074 | -0.294 | -0.071 | -0.272 | -0.067 | -0.195 | -0.083 | -0.286 | -0.079 | -0.316 | -0.100 | -0.345 | -0.140 | -0.552 | -0.142 | -0.530 | -0.153 | -0.471 |
| $\mathbf{ES_{0.95}}$ | | | | | | | | | | | | | | | | | | |
| $T = 75$ | -0.381 | -0.676 | -0.347 | -0.597 | -0.289 | -0.426 | -0.451 | -0.637 | -0.414 | -0.649 | -0.388 | -0.632 | -0.712 | -1.067 | -0.657 | -0.994 | -0.542 | -0.830 |
| $T = 250$ | -0.212 | -0.506 | -0.203 | -0.437 | -0.190 | -0.295 | -0.254 | -0.469 | -0.245 | -0.507 | -0.270 | -0.551 | -0.437 | -0.949 | -0.434 | -0.907 | -0.413 | -0.800 |
| $T = 2500$ | -0.068 | -0.418 | -0.067 | -0.350 | -0.070 | -0.226 | -0.080 | -0.366 | -0.079 | -0.418 | -0.098 | -0.507 | -0.139 | -0.916 | -0.141 | -0.877 | -0.153 | -0.806 |
| $\mathbf{MVaR_{0.95}}$ | | | | | | | | | | | | | | | | | | |
| $T = 75$ | -0.350 | -0.589 | -0.307 | -0.547 | -0.229 | -0.449 | -0.418 | -0.576 | -0.376 | -0.589 | -0.321 | -0.547 | -0.545 | -0.756 | -0.478 | -0.693 | -0.352 | -0.543 |
| $T = 250$ | -0.193 | -0.436 | -0.177 | -0.405 | -0.146 | -0.342 | -0.235 | -0.420 | -0.219 | -0.455 | -0.204 | -0.467 | -0.324 | -0.596 | -0.295 | -0.558 | -0.223 | -0.450 |
| $T = 2500$ | -0.061 | -0.355 | -0.058 | -0.331 | -0.054 | -0.289 | -0.074 | -0.323 | -0.070 | -0.374 | -0.071 | -0.432 | -0.102 | -0.516 | -0.094 | -0.478 | -0.077 | -0.399 |
| $\mathbf{EVaR_{0.95}}$ | | | | | | | | | | | | | | | | | | |
| $T = 75$ | -0.323 | -0.441 | -0.284 | -0.402 | -0.202 | -0.309 | -0.377 | -0.447 | -0.336 | -0.445 | -0.275 | -0.394 | -0.476 | -0.597 | -0.425 | -0.549 | -0.323 | -0.433 |
| $T = 250$ | -0.178 | -0.292 | -0.160 | -0.264 | -0.124 | -0.193 | -0.211 | -0.291 | -0.192 | -0.306 | -0.173 | -0.295 | -0.279 | -0.443 | -0.260 | -0.416 | -0.212 | -0.338 |
| $T = 2500$ | -0.057 | -0.200 | -0.052 | -0.181 | -0.044 | -0.128 | -0.066 | -0.191 | -0.061 | -0.217 | -0.059 | -0.241 | -0.088 | -0.359 | -0.082 | -0.336 | -0.075 | -0.287 |

Table S.3: Relative frequency of risk underrating for Poi-, ZIP, and NB-INAR(1) DGPs, coherent ("Coh") vs. approximate ("Gau") forecasts, for different $(\alpha, T)$, $I = 2.4$ in case of overdispersion and forecast horizon $h = 1$, computed across all simulation runs and all levels of $\mu$.

| | Poi-INAR(1) DGP | | | | | | ZIP-INAR(1) DGP | | | | | | NB-INAR(1) DGP | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\alpha = 0.33$ | | $\alpha = 0.55$ | | $\alpha = 0.80$ | | $\alpha = 0.33$ | | $\alpha = 0.55$ | | $\alpha = 0.80$ | | $\alpha = 0.33$ | | $\alpha = 0.55$ | | $\alpha = 0.80$ | |
| | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau |
| $\mathbf{VaR_{0.95}}$ | | | | | | | | | | | | | | | | | | |
| $T = 75$ | 0.171 | 0.631 | 0.142 | 0.587 | 0.076 | 0.485 | 0.269 | 0.572 | 0.293 | 0.708 | 0.383 | 0.850 | 0.388 | 0.797 | 0.397 | 0.775 | 0.365 | 0.674 |
| $T = 250$ | 0.099 | 0.663 | 0.086 | 0.608 | 0.061 | 0.498 | 0.158 | 0.574 | 0.176 | 0.769 | 0.240 | 0.947 | 0.261 | 0.897 | 0.266 | 0.872 | 0.249 | 0.747 |
| $T = 2500$ | 0.034 | 0.685 | 0.030 | 0.619 | 0.026 | 0.505 | 0.051 | 0.575 | 0.057 | 0.813 | 0.072 | 0.990 | 0.083 | 0.988 | 0.086 | 0.968 | 0.078 | 0.828 |
| $\mathbf{TCE_{0.95}}$ | | | | | | | | | | | | | | | | | | |
| $T = 75$ | 0.475 | 0.527 | 0.408 | 0.458 | 0.284 | 0.328 | 0.524 | 0.423 | 0.542 | 0.589 | 0.656 | 0.841 | 0.579 | 0.916 | 0.598 | 0.920 | 0.624 | 0.894 |
| $T = 250$ | 0.482 | 0.525 | 0.443 | 0.435 | 0.373 | 0.299 | 0.513 | 0.388 | 0.527 | 0.604 | 0.600 | 0.936 | 0.548 | 0.987 | 0.567 | 0.988 | 0.603 | 0.970 |
| $T = 2500$ | 0.495 | 0.526 | 0.480 | 0.418 | 0.461 | 0.281 | 0.503 | 0.368 | 0.509 | 0.604 | 0.534 | 0.985 | 0.513 | 1.000 | 0.524 | 1.000 | 0.541 | 0.992 |
| $\mathbf{ES_{0.95}}$ | | | | | | | | | | | | | | | | | | |
| $T = 75$ | 0.495 | 0.758 | 0.440 | 0.697 | 0.308 | 0.603 | 0.523 | 0.558 | 0.537 | 0.716 | 0.628 | 0.919 | 0.572 | 0.959 | 0.592 | 0.961 | 0.630 | 0.952 |
| $T = 250$ | 0.497 | 0.861 | 0.464 | 0.779 | 0.388 | 0.682 | 0.514 | 0.549 | 0.522 | 0.768 | 0.580 | 0.987 | 0.541 | 0.998 | 0.557 | 0.998 | 0.591 | 0.997 |
| $T = 2500$ | 0.499 | 0.964 | 0.489 | 0.863 | 0.466 | 0.763 | 0.504 | 0.533 | 0.506 | 0.801 | 0.525 | 1.000 | 0.511 | 1.000 | 0.518 | 1.000 | 0.533 | 1.000 |
| $\mathbf{MVaR_{0.95}}$ | | | | | | | | | | | | | | | | | | |
| $T = 75$ | 0.500 | 0.752 | 0.456 | 0.727 | 0.333 | 0.719 | 0.524 | 0.618 | 0.537 | 0.756 | 0.601 | 0.888 | 0.562 | 0.826 | 0.572 | 0.807 | 0.559 | 0.723 |
| $T = 250$ | 0.499 | 0.854 | 0.474 | 0.820 | 0.404 | 0.829 | 0.515 | 0.640 | 0.523 | 0.837 | 0.565 | 0.978 | 0.534 | 0.939 | 0.547 | 0.919 | 0.551 | 0.826 |
| $T = 2500$ | 0.500 | 0.962 | 0.492 | 0.910 | 0.470 | 0.928 | 0.505 | 0.639 | 0.505 | 0.900 | 0.522 | 0.997 | 0.509 | 1.000 | 0.514 | 0.999 | 0.516 | 0.931 |
| $\mathbf{EVaR_{0.95}}$ | | | | | | | | | | | | | | | | | | |
| $T = 75$ | 0.501 | 0.661 | 0.467 | 0.626 | 0.362 | 0.556 | 0.518 | 0.580 | 0.527 | 0.700 | 0.587 | 0.847 | 0.557 | 0.841 | 0.572 | 0.846 | 0.599 | 0.836 |
| $T = 250$ | 0.500 | 0.748 | 0.480 | 0.690 | 0.420 | 0.610 | 0.512 | 0.602 | 0.516 | 0.782 | 0.555 | 0.962 | 0.532 | 0.951 | 0.545 | 0.955 | 0.573 | 0.952 |
| $T = 2500$ | 0.500 | 0.891 | 0.494 | 0.788 | 0.477 | 0.682 | 0.504 | 0.597 | 0.502 | 0.862 | 0.518 | 1.000 | 0.509 | 1.000 | 0.514 | 1.000 | 0.526 | 1.000 |

Table S.4: Mean severity of risk underrating for Poi-, ZIP, and NB-INAR(1) DGPs, coherent ("Coh") vs. approximate ("Gau") forecasts, for different $(\alpha, T)$, $I = 2.4$ in case of overdispersion and forecast horizon $h = 1$, computed across all simulation runs and all levels of $\mu$.

| | Poi-INAR(1) DGP | | | | | | ZIP-INAR(1) DGP | | | | | | NB-INAR(1) DGP | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\alpha = 0.33$ | | $\alpha = 0.55$ | | $\alpha = 0.80$ | | $\alpha = 0.33$ | | $\alpha = 0.55$ | | $\alpha = 0.80$ | | $\alpha = 0.33$ | | $\alpha = 0.55$ | | $\alpha = 0.80$ | |
| | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau |
| $\mathbf{VaR_{0.95}}$ | | | | | | | | | | | | | | | | | | |
| $T = 75$ | -1.034 | -0.582 | -1.020 | -0.538 | -1.005 | -0.424 | -1.133 | -0.777 | -1.148 | -0.926 | -1.244 | -1.066 | -1.384 | -1.283 | -1.365 | -1.240 | -1.247 | -0.985 |
| $T = 250$ | -1.002 | -0.444 | -1.001 | -0.416 | -1.000 | -0.332 | -1.016 | -0.599 | -1.017 | -0.734 | -1.068 | -0.915 | -1.084 | -0.984 | -1.074 | -0.939 | -1.042 | -0.737 |
| $T = 2500$ | -1.000 | -0.378 | -1.000 | -0.357 | -1.000 | -0.292 | -1.000 | -0.520 | -1.000 | -0.641 | -1.004 | -0.865 | -1.000 | -0.844 | -1.000 | -0.787 | -1.000 | -0.578 |
| $\mathbf{TCE_{0.95}}$ | | | | | | | | | | | | | | | | | | |
| $T = 75$ | -0.402 | -0.553 | -0.386 | -0.499 | -0.304 | -0.371 | -0.622 | -0.776 | -0.681 | -0.948 | -0.934 | -1.216 | -1.339 | -1.959 | -1.427 | -2.003 | -1.431 | -1.773 |
| $T = 250$ | -0.224 | -0.384 | -0.219 | -0.352 | -0.190 | -0.256 | -0.336 | -0.607 | -0.376 | -0.750 | -0.555 | -1.043 | -0.757 | -1.721 | -0.810 | -1.777 | -0.868 | -1.597 |
| $T = 2500$ | -0.074 | -0.294 | -0.071 | -0.272 | -0.067 | -0.195 | -0.109 | -0.542 | -0.123 | -0.663 | -0.178 | -0.979 | -0.242 | -1.674 | -0.266 | -1.731 | -0.299 | -1.539 |
| $\mathbf{ES_{0.95}}$ | | | | | | | | | | | | | | | | | | |
| $T = 75$ | -0.381 | -0.676 | -0.347 | -0.597 | -0.289 | -0.426 | -0.619 | -0.899 | -0.676 | -1.100 | -0.894 | -1.443 | -1.349 | -2.317 | -1.433 | -2.371 | -1.427 | -2.150 |
| $T = 250$ | -0.212 | -0.506 | -0.203 | -0.437 | -0.190 | -0.295 | -0.335 | -0.726 | -0.369 | -0.899 | -0.521 | -1.330 | -0.755 | -2.157 | -0.804 | -2.222 | -0.861 | -2.052 |
| $T = 2500$ | -0.068 | -0.418 | -0.067 | -0.350 | -0.070 | -0.226 | -0.105 | -0.663 | -0.116 | -0.801 | -0.167 | -1.315 | -0.239 | -2.134 | -0.256 | -2.197 | -0.272 | -2.029 |
| $\mathbf{MVaR_{0.95}}$ | | | | | | | | | | | | | | | | | | |
| $T = 75$ | -0.350 | -0.589 | -0.307 | -0.547 | -0.229 | -0.449 | -0.576 | -0.794 | -0.614 | -0.941 | -0.738 | -1.090 | -0.945 | -1.301 | -0.925 | -1.254 | -0.772 | -0.995 |
| $T = 250$ | -0.193 | -0.436 | -0.177 | -0.405 | -0.146 | -0.342 | -0.312 | -0.609 | -0.332 | -0.746 | -0.426 | -0.962 | -0.525 | -1.006 | -0.510 | -0.953 | -0.452 | -0.738 |
| $T = 2500$ | -0.061 | -0.355 | -0.058 | -0.331 | -0.054 | -0.289 | -0.098 | -0.531 | -0.104 | -0.650 | -0.134 | -0.942 | -0.165 | -0.911 | -0.161 | -0.836 | -0.145 | -0.587 |
| $\mathbf{EVaR_{0.95}}$ | | | | | | | | | | | | | | | | | | |
| $T = 75$ | -0.323 | -0.441 | -0.284 | -0.402 | -0.202 | -0.309 | -0.505 | -0.617 | -0.514 | -0.713 | -0.561 | -0.807 | -0.809 | -1.100 | -0.816 | -1.096 | -0.745 | -0.962 |
| $T = 250$ | -0.178 | -0.292 | -0.160 | -0.264 | -0.124 | -0.193 | -0.275 | -0.429 | -0.277 | -0.520 | -0.311 | -0.662 | -0.449 | -0.866 | -0.451 | -0.870 | -0.437 | -0.778 |
| $T = 2500$ | -0.057 | -0.200 | -0.052 | -0.181 | -0.044 | -0.128 | -0.086 | -0.340 | -0.086 | -0.416 | -0.096 | -0.630 | -0.141 | -0.800 | -0.143 | -0.806 | -0.138 | -0.717 |

## Section 4.2 Further Autoregressive Count DGPs

Table S.5: Relative frequency of risk underrating for Poisson INAR(2), INARCH(1) and INARCH(2) DGPs, coherent ("Coh") vs. approximate ("Gau") forecasts, for different $(\alpha, T)$, $\alpha_2 = 0.25$ in case of of the second-order models and forecast horizon $h = 1$, computed across all simulation runs and all levels of $\mu$.

| | Poi-INAR(2) DGP | | | | | | Poi-INARCH(1) DGP | | | | | | Poi-INARCH(2) DGP | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\alpha = 0.33$ | | $\alpha = 0.55$ | | $\alpha = 0.80$ | | $\alpha = 0.33$ | | $\alpha = 0.55$ | | $\alpha = 0.80$ | | $\alpha = 0.33$ | | $\alpha = 0.55$ | | $\alpha = 0.80$ | |
| | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau |
| $\mathbf{VaR_{0.95}}$ | | | | | | | | | | | | | | | | | | |
| $T = 75$ | 0.201 | 0.601 | 0.188 | 0.559 | 0.178 | 0.487 | 0.187 | 0.630 | 0.192 | 0.589 | 0.199 | 0.502 | 0.224 | 0.613 | 0.232 | 0.573 | 0.250 | 0.506 |
| $T = 250$ | 0.115 | 0.629 | 0.109 | 0.572 | 0.105 | 0.480 | 0.109 | 0.664 | 0.110 | 0.598 | 0.112 | 0.489 | 0.131 | 0.643 | 0.137 | 0.585 | 0.149 | 0.492 |
| $T = 2500$ | 0.037 | 0.651 | 0.036 | 0.582 | 0.035 | 0.482 | 0.033 | 0.696 | 0.033 | 0.602 | 0.031 | 0.490 | 0.042 | 0.664 | 0.044 | 0.593 | 0.047 | 0.488 |
| $\mathbf{TCE_{0.95}}$ | | | | | | | | | | | | | | | | | | |
| $T = 75$ | 0.477 | 0.501 | 0.439 | 0.452 | 0.369 | 0.388 | 0.494 | 0.555 | 0.507 | 0.524 | 0.484 | 0.461 | 0.505 | 0.545 | 0.504 | 0.516 | 0.496 | 0.469 |
| $T = 250$ | 0.483 | 0.493 | 0.460 | 0.425 | 0.412 | 0.353 | 0.505 | 0.564 | 0.498 | 0.510 | 0.498 | 0.444 | 0.505 | 0.548 | 0.501 | 0.508 | 0.495 | 0.447 |
| $T = 2500$ | 0.493 | 0.487 | 0.487 | 0.410 | 0.468 | 0.336 | 0.505 | 0.574 | 0.504 | 0.515 | 0.492 | 0.439 | 0.500 | 0.551 | 0.500 | 0.504 | 0.496 | 0.438 |
| $\mathbf{ES_{0.95}}$ | | | | | | | | | | | | | | | | | | |
| $T = 75$ | 0.496 | 0.702 | 0.471 | 0.629 | 0.421 | 0.521 | 0.494 | 0.757 | 0.507 | 0.678 | 0.484 | 0.550 | 0.505 | 0.715 | 0.504 | 0.647 | 0.496 | 0.549 |
| $T = 250$ | 0.496 | 0.781 | 0.483 | 0.668 | 0.450 | 0.514 | 0.505 | 0.831 | 0.498 | 0.700 | 0.498 | 0.551 | 0.505 | 0.781 | 0.501 | 0.677 | 0.495 | 0.541 |
| $T = 2500$ | 0.498 | 0.859 | 0.495 | 0.698 | 0.484 | 0.518 | 0.505 | 0.898 | 0.504 | 0.728 | 0.492 | 0.550 | 0.500 | 0.833 | 0.500 | 0.696 | 0.496 | 0.540 |
| $\mathbf{MVaR_{0.95}}$ | | | | | | | | | | | | | | | | | | |
| $T = 75$ | 0.501 | 0.707 | 0.484 | 0.661 | 0.451 | 0.590 | 0.494 | 0.734 | 0.507 | 0.671 | 0.484 | 0.551 | 0.505 | 0.698 | 0.504 | 0.639 | 0.496 | 0.552 |
| $T = 250$ | 0.500 | 0.793 | 0.491 | 0.719 | 0.469 | 0.608 | 0.505 | 0.815 | 0.498 | 0.696 | 0.498 | 0.554 | 0.505 | 0.770 | 0.501 | 0.673 | 0.495 | 0.540 |
| $T = 2500$ | 0.499 | 0.886 | 0.497 | 0.772 | 0.492 | 0.628 | 0.505 | 0.901 | 0.504 | 0.722 | 0.492 | 0.548 | 0.500 | 0.832 | 0.500 | 0.697 | 0.496 | 0.541 |
| $\mathbf{EVaR_{0.95}}$ | | | | | | | | | | | | | | | | | | |
| $T = 75$ | 0.502 | 0.623 | 0.487 | 0.577 | 0.461 | 0.508 | 0.494 | 0.659 | 0.507 | 0.611 | 0.484 | 0.516 | 0.505 | 0.631 | 0.504 | 0.586 | 0.496 | 0.516 |
| $T = 250$ | 0.501 | 0.687 | 0.492 | 0.610 | 0.474 | 0.504 | 0.505 | 0.735 | 0.498 | 0.635 | 0.498 | 0.516 | 0.505 | 0.688 | 0.501 | 0.612 | 0.495 | 0.507 |
| $T = 2500$ | 0.499 | 0.781 | 0.497 | 0.655 | 0.495 | 0.513 | 0.505 | 0.821 | 0.504 | 0.665 | 0.492 | 0.519 | 0.500 | 0.755 | 0.500 | 0.638 | 0.496 | 0.507 |

Table S.6: Mean severity of risk underrating for Poisson INAR(2), INARCH(1) and INARCH(2) DGPs, coherent ("Coh") vs. approximate ("Gau") forecasts, for different $(\alpha, T)$, $\alpha_2 = 0.25$ in case of of the second-order models and forecast horizon $h = 1$, computed across all simulation runs and all levels of $\mu$.

| | Poi-INAR(2) DGP | | | | | | Poi-INARCH(1) DGP | | | | | | Poi-INARCH(2) DGP | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\alpha = 0.33$ | | $\alpha = 0.55$ | | $\alpha = 0.80$ | | $\alpha = 0.33$ | | $\alpha = 0.55$ | | $\alpha = 0.80$ | | $\alpha = 0.33$ | | $\alpha = 0.55$ | | $\alpha = 0.80$ | |
| | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau |
| $\mathbf{VaR_{0.95}}$ | | | | | | | | | | | | | | | | | | |
| $T = 75$ | -1.070 | -0.653 | -1.070 | -0.675 | -1.122 | -0.749 | -1.061 | -0.660 | -1.104 | -0.804 | -1.206 | -1.221 | -1.112 | -0.749 | -1.148 | -0.904 | -1.251 | -1.329 |
| $T = 250$ | -1.009 | -0.489 | -1.007 | -0.515 | -1.018 | -0.600 | -1.005 | -0.520 | -1.005 | -0.648 | -1.037 | -1.074 | -1.017 | -0.572 | -1.025 | -0.717 | -1.057 | -1.132 |
| $T = 2500$ | -1.000 | -0.406 | -1.000 | -0.436 | -1.000 | -0.522 | -1.000 | -0.434 | -1.000 | -0.584 | -1.000 | -0.955 | -1.000 | -0.481 | -1.000 | -0.626 | -1.000 | -1.016 |
| $\mathbf{TCE_{0.95}}$ | | | | | | | | | | | | | | | | | | |
| $T = 75$ | -0.479 | -0.634 | -0.485 | -0.657 | -0.568 | -0.731 | -0.428 | -0.663 | -0.445 | -0.858 | -0.529 | -1.380 | -0.526 | -0.768 | -0.564 | -0.967 | -0.669 | -1.482 |
| $T = 250$ | -0.258 | -0.445 | -0.258 | -0.487 | -0.280 | -0.594 | -0.231 | -0.501 | -0.237 | -0.695 | -0.251 | -1.239 | -0.283 | -0.574 | -0.299 | -0.774 | -0.340 | -1.298 |
| $T = 2500$ | -0.082 | -0.343 | -0.081 | -0.398 | -0.083 | -0.515 | -0.069 | -0.401 | -0.071 | -0.614 | -0.069 | -1.118 | -0.090 | -0.470 | -0.093 | -0.678 | -0.100 | -1.189 |
| $\mathbf{ES_{0.95}}$ | | | | | | | | | | | | | | | | | | |
| $T = 75$ | -0.458 | -0.748 | -0.446 | -0.760 | -0.490 | -0.828 | -0.430 | -0.798 | -0.446 | -0.990 | -0.542 | -1.515 | -0.526 | -0.903 | -0.564 | -1.100 | -0.674 | -1.613 |
| $T = 250$ | -0.249 | -0.554 | -0.241 | -0.575 | -0.253 | -0.673 | -0.229 | -0.633 | -0.236 | -0.818 | -0.267 | -1.348 | -0.286 | -0.707 | -0.299 | -0.897 | -0.343 | -1.413 |
| $T = 2500$ | -0.079 | -0.450 | -0.077 | -0.487 | -0.078 | -0.589 | -0.072 | -0.548 | -0.073 | -0.742 | -0.082 | -1.245 | -0.090 | -0.605 | -0.092 | -0.800 | -0.103 | -1.300 |
| $\mathbf{MVaR_{0.95}}$ | | | | | | | | | | | | | | | | | | |
| $T = 75$ | -0.426 | -0.661 | -0.412 | -0.685 | -0.436 | -0.761 | -0.398 | -0.665 | -0.415 | -0.808 | -0.505 | -1.233 | -0.486 | -0.757 | -0.521 | -0.913 | -0.625 | -1.326 |
| $T = 250$ | -0.231 | -0.481 | -0.221 | -0.508 | -0.223 | -0.598 | -0.211 | -0.508 | -0.219 | -0.645 | -0.247 | -1.058 | -0.265 | -0.567 | -0.277 | -0.715 | -0.318 | -1.125 |
| $T = 2500$ | -0.073 | -0.382 | -0.069 | -0.421 | -0.067 | -0.513 | -0.066 | -0.422 | -0.068 | -0.577 | -0.075 | -0.968 | -0.083 | -0.467 | -0.085 | -0.617 | -0.094 | -1.005 |
| $\mathbf{EVaR_{0.95}}$ | | | | | | | | | | | | | | | | | | |
| $T = 75$ | -0.397 | -0.514 | -0.386 | -0.532 | -0.396 | -0.592 | -0.367 | -0.503 | -0.384 | -0.615 | -0.468 | -0.949 | -0.449 | -0.591 | -0.483 | -0.713 | -0.579 | -1.044 |
| $T = 250$ | -0.216 | -0.338 | -0.207 | -0.359 | -0.202 | -0.428 | -0.195 | -0.350 | -0.202 | -0.454 | -0.228 | -0.770 | -0.245 | -0.409 | -0.256 | -0.519 | -0.294 | -0.828 |
| $T = 2500$ | -0.068 | -0.232 | -0.065 | -0.263 | -0.060 | -0.334 | -0.061 | -0.267 | -0.063 | -0.384 | -0.069 | -0.666 | -0.077 | -0.304 | -0.079 | -0.418 | -0.087 | -0.702 |

**Note:** Given $(\alpha, T)$, the frequencies of risk underrating for the coherent TCE, ES, MVaR, and EVaR forecasts agree with each other in case of the Poi-INARCH models, see Tables S.5, S.7, and S.9. This is explained by the fact that the 1-step-ahead forecast distribution of these models is a one-parameter Poi-distribution: since all risk measures are strictly increasing in the Poisson's mean, we have a unique type of deviation from the true risk value.

Table S.7: Relative frequency of risk underrating for Poisson INAR(2), INARCH(1) and INARCH(2) DGPs, coherent ("Coh") vs. approximate ("Gau") forecasts, for different $(\alpha, T)$, $\alpha_2 = 0.35$ in case of of the second-order models and forecast horizon $h = 1$, computed across all simulation runs and all levels of $\mu$.

| | Poi-INAR(2) DGP | | | | | | Poi-INARCH(1) DGP | | | | | | Poi-INARCH(2) DGP | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\alpha = 0.33$ | | $\alpha = 0.55$ | | $\alpha = 0.80$ | | $\alpha = 0.33$ | | $\alpha = 0.55$ | | $\alpha = 0.80$ | | $\alpha = 0.33$ | | $\alpha = 0.55$ | | $\alpha = 0.80$ | |
| | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau |
| $\mathbf{VaR_{0.95}}$ | | | | | | | | | | | | | | | | | | |
| $T = 75$ | 0.187 | 0.584 | 0.176 | 0.543 | 0.165 | 0.479 | 0.187 | 0.630 | 0.192 | 0.589 | 0.199 | 0.502 | 0.227 | 0.599 | 0.234 | 0.565 | 0.254 | 0.505 |
| $T = 250$ | 0.112 | 0.609 | 0.102 | 0.548 | 0.097 | 0.467 | 0.109 | 0.664 | 0.110 | 0.598 | 0.112 | 0.489 | 0.135 | 0.623 | 0.141 | 0.571 | 0.151 | 0.486 |
| $T = 2500$ | 0.036 | 0.630 | 0.035 | 0.563 | 0.031 | 0.466 | 0.033 | 0.696 | 0.033 | 0.602 | 0.031 | 0.490 | 0.043 | 0.643 | 0.043 | 0.579 | 0.047 | 0.479 |
| $\mathbf{TCE_{0.95}}$ | | | | | | | | | | | | | | | | | | |
| $T = 75$ 0.433 | 0.479 | 0.405 | 0.436 | 0.338 | 0.384 | 0.494 | 0.555 | 0.507 | 0.524 | 0.484 | 0.461 | 0.503 | 0.534 | 0.503 | 0.512 | 0.497 | 0.470 | |
| $T = 250$ | 0.458 | 0.466 | 0.437 | 0.406 | 0.393 | 0.350 | 0.505 | 0.564 | 0.498 | 0.510 | 0.498 | 0.444 | 0.504 | 0.533 | 0.502 | 0.501 | 0.491 | 0.444 |
| $T = 2500$ | 0.486 | 0.456 | 0.480 | 0.394 | 0.458 | 0.330 | 0.505 | 0.574 | 0.504 | 0.515 | 0.492 | 0.439 | 0.500 | 0.537 | 0.502 | 0.496 | 0.495 | 0.434 |
| $\mathbf{ES_{0.95}}$ | | | | | | | | | | | | | | | | | | |
| $T = 75$ | 0.433 | 0.479 | 0.405 | 0.436 | 0.338 | 0.384 | 0.494 | 0.555 | 0.507 | 0.524 | 0.484 | 0.461 | 0.503 | 0.534 | 0.503 | 0.512 | 0.497 | 0.470 |
| $T = 250$ | 0.458 | 0.466 | 0.437 | 0.406 | 0.393 | 0.350 | 0.505 | 0.564 | 0.498 | 0.510 | 0.498 | 0.444 | 0.504 | 0.533 | 0.502 | 0.501 | 0.491 | 0.444 |
| $T = 2500$ | 0.486 | 0.456 | 0.480 | 0.394 | 0.458 | 0.330 | 0.505 | 0.574 | 0.504 | 0.515 | 0.492 | 0.439 | 0.500 | 0.537 | 0.502 | 0.496 | 0.495 | 0.434 |
| $\mathbf{MVaR_{0.95}}$ | | | | | | | | | | | | | | | | | | |
| $T = 75$ | 0.478 | 0.690 | 0.466 | 0.642 | 0.435 | 0.568 | 0.494 | 0.734 | 0.507 | 0.671 | 0.484 | 0.551 | 0.503 | 0.677 | 0.503 | 0.628 | 0.497 | 0.552 |
| $T = 250$ | 0.489 | 0.771 | 0.477 | 0.691 | 0.462 | 0.576 | 0.505 | 0.815 | 0.498 | 0.696 | 0.498 | 0.554 | 0.504 | 0.734 | 0.502 | 0.654 | 0.491 | 0.535 |
| $T = 2500$ | 0.497 | 0.852 | 0.494 | 0.740 | 0.483 | 0.586 | 0.505 | 0.901 | 0.504 | 0.722 | 0.492 | 0.548 | 0.500 | 0.787 | 0.502 | 0.680 | 0.495 | 0.530 |
| $\mathbf{EVaR_{0.95}}$ | | | | | | | | | | | | | | | | | | |
| $T = 75$ | 0.483 | 0.605 | 0.473 | 0.560 | 0.445 | 0.496 | 0.494 | 0.659 | 0.507 | 0.611 | 0.484 | 0.516 | 0.503 | 0.614 | 0.503 | 0.578 | 0.497 | 0.518 |
| $T = 250$ | 0.492 | 0.661 | 0.481 | 0.584 | 0.467 | 0.487 | 0.505 | 0.735 | 0.498 | 0.635 | 0.498 | 0.516 | 0.504 | 0.660 | 0.502 | 0.598 | 0.491 | 0.502 |
| $T = 2500$ | 0.498 | 0.741 | 0.496 | 0.619 | 0.485 | 0.487 | 0.505 | 0.821 | 0.504 | 0.665 | 0.492 | 0.519 | 0.500 | 0.716 | 0.502 | 0.624 | 0.495 | 0.499 |

Table S.8: Mean severity of risk underrating for Poisson INAR(2), INARCH(1) and INARCH(2) DGPs, coherent ("Coh") vs. approximate ("Gau") forecasts, for different $(\alpha, T)$, $\alpha_2 = 0.35$ in case of of the second-order models and forecast horizon $h = 1$, computed across all simulation runs and all levels of $\mu$.

| | Poi-INAR(2) DGP | | | | | | Poi-INARCH(1) DGP | | | | | | Poi-INARCH(2) DGP | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\alpha = 0.33$ | | $\alpha = 0.55$ | | $\alpha = 0.80$ | | $\alpha = 0.33$ | | $\alpha = 0.55$ | | $\alpha = 0.80$ | | $\alpha = 0.33$ | | $\alpha = 0.55$ | | $\alpha = 0.80$ | |
| | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau |
| $\mathbf{VaR_{0.95}}$ | | | | | | | | | | | | | | | | | | |
| $T = 75$ | -1.063 | -0.653 | -1.071 | -0.698 | -1.128 | -0.816 | -1.061 | -0.660 | -1.104 | -0.804 | -1.206 | -1.221 | -1.119 | -0.795 | -1.164 | -0.955 | -1.275 | -1.377 |
| $T = 250$ | -1.007 | -0.491 | -1.005 | -0.533 | -1.013 | -0.667 | -1.005 | -0.520 | -1.005 | -0.648 | -1.037 | -1.074 | -1.019 | -0.613 | -1.029 | -0.769 | -1.066 | -1.177 |
| $T = 2500$ | -1.000 | -0.406 | -1.000 | -0.453 | -1.000 | -0.580 | -1.000 | -0.434 | -1.000 | -0.584 | -1.000 | -0.955 | -1.000 | -0.519 | -1.000 | -0.664 | -1.001 | -1.052 |
| $\mathbf{TCE_{0.95}}$ | | | | | | | | | | | | | | | | | | |
| $T = 75$ | -0.480 | -0.632 | -0.479 | -0.683 | -0.558 | -0.806 | -0.428 | -0.663 | -0.445 | -0.858 | -0.529 | -1.380 | -0.538 | -0.828 | -0.578 | -1.027 | -0.694 | -1.534 |
| $T = 250$ | -0.262 | -0.449 | -0.250 | -0.507 | -0.260 | -0.671 | -0.231 | -0.501 | -0.237 | -0.695 | -0.251 | -1.239 | -0.290 | -0.630 | -0.307 | -0.836 | -0.348 | -1.353 |
| $T = 2500$ | -0.080 | -0.348 | -0.078 | -0.418 | -0.074 | -0.596 | -0.069 | -0.401 | -0.071 | -0.614 | -0.069 | -1.118 | -0.092 | -0.527 | -0.092 | -0.729 | -0.102 | -1.228 |
| $\mathbf{ES_{0.95}}$ | | | | | | | | | | | | | | | | | | |
| $T = 75$ | -0.439 | -0.741 | -0.425 | -0.782 | -0.462 | -0.909 | -0.430 | -0.798 | -0.446 | -0.990 | -0.542 | -1.515 | -0.539 | -0.960 | -0.580 | -1.158 | -0.701 | -1.666 |
| $T = 250$ | -0.243 | -0.548 | -0.230 | -0.595 | -0.227 | -0.758 | -0.229 | -0.633 | -0.236 | -0.818 | -0.267 | -1.348 | -0.290 | -0.759 | -0.307 | -0.956 | -0.352 | -1.469 |
| $T = 2500$ | -0.078 | -0.446 | -0.075 | -0.503 | -0.072 | -0.677 | -0.072 | -0.548 | -0.073 | -0.742 | -0.082 | -1.245 | -0.090 | -0.658 | -0.093 | -0.854 | -0.104 | -1.344 |
| $\mathbf{MVaR_{0.95}}$ | | | | | | | | | | | | | | | | | | |
| $T = 75$ | -0.411 | -0.661 | -0.401 | -0.708 | -0.431 | -0.831 | -0.398 | -0.665 | -0.415 | -0.808 | -0.505 | -1.233 | -0.498 | -0.802 | -0.536 | -0.962 | -0.652 | -1.370 |
| $T = 250$ | -0.225 | -0.482 | -0.212 | -0.525 | -0.210 | -0.667 | -0.211 | -0.508 | -0.219 | -0.645 | -0.247 | -1.058 | -0.268 | -0.608 | -0.283 | -0.761 | -0.327 | -1.166 |
| $T = 2500$ | -0.072 | -0.387 | -0.068 | -0.436 | -0.065 | -0.578 | -0.066 | -0.422 | -0.068 | -0.577 | -0.075 | -0.968 | -0.084 | -0.508 | -0.086 | -0.652 | -0.096 | -1.040 |
| $\mathbf{EVaR_{0.95}}$ | | | | | | | | | | | | | | | | | | |
| $T = 75$ | -0.387 | -0.515 | -0.380 | -0.552 | -0.403 | -0.651 | -0.367 | -0.503 | -0.384 | -0.615 | -0.468 | -0.949 | -0.461 | -0.628 | -0.496 | -0.753 | -0.603 | -1.082 |
| $T = 250$ | -0.211 | -0.340 | -0.200 | -0.372 | -0.198 | -0.481 | -0.195 | -0.350 | -0.202 | -0.454 | -0.228 | -0.770 | -0.248 | -0.440 | -0.262 | -0.555 | -0.302 | -0.862 |
| $T = 2500$ | -0.066 | -0.235 | -0.063 | -0.277 | -0.060 | -0.383 | -0.061 | -0.267 | -0.063 | -0.384 | -0.069 | -0.666 | -0.077 | -0.335 | -0.080 | -0.444 | -0.088 | -0.726 |

Table S.9: Relative frequency of risk underrating for Poisson INAR(2), INARCH(1) and INARCH(2) DGPs, coherent ("Coh") vs. approximate ("Gau") forecasts, for different $(\alpha, T)$, $\alpha_2 = 0.45$ in case of of the second-order models and forecast horizon $h = 1$, computed across all simulation runs and all levels of $\mu$.

| | Poi-INAR(2) DGP | | | | | | Poi-INARCH(1) DGP | | | | | | Poi-INARCH(2) DGP | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\alpha = 0.33$ | | $\alpha = 0.55$ | | $\alpha = 0.80$ | | $\alpha = 0.33$ | | $\alpha = 0.55$ | | $\alpha = 0.80$ | | $\alpha = 0.33$ | | $\alpha = 0.55$ | | $\alpha = 0.80$ | |
| | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau |
| $\mathbf{VaR_{0.95}}$ | | | | | | | | | | | | | | | | | | |
| $T = 75$ | 0.165 | 0.561 | 0.157 | 0.527 | 0.150 | 0.472 | 0.187 | 0.630 | 0.192 | 0.589 | 0.199 | 0.502 | 0.230 | 0.581 | 0.238 | 0.549 | 0.257 | 0.502 |
| $T = 250$ | 0.102 | 0.585 | 0.095 | 0.530 | 0.087 | 0.459 | 0.109 | 0.664 | 0.110 | 0.598 | 0.112 | 0.489 | 0.138 | 0.603 | 0.139 | 0.554 | 0.153 | 0.480 |
| $T = 2500$ | 0.034 | 0.607 | 0.033 | 0.542 | 0.030 | 0.452 | 0.033 | 0.696 | 0.033 | 0.602 | 0.031 | 0.490 | 0.044 | 0.618 | 0.045 | 0.562 | 0.047 | 0.474 |
| $\mathbf{TCE_{0.95}}$ | | | | | | | | | | | | | | | | | | |
| $T = 75$ | 0.382 | 0.449 | 0.354 | 0.419 | 0.296 | 0.379 | 0.494 | 0.555 | 0.507 | 0.524 | 0.484 | 0.461 | 0.504 | 0.520 | 0.501 | 0.501 | 0.491 | 0.468 |
| $T = 250$ | 0.425 | 0.432 | 0.405 | 0.389 | 0.355 | 0.346 | 0.505 | 0.564 | 0.498 | 0.510 | 0.498 | 0.444 | 0.502 | 0.520 | 0.503 | 0.490 | 0.488 | 0.442 |
| $T = 2500$ | 0.474 | 0.421 | 0.464 | 0.374 | 0.442 | 0.327 | 0.505 | 0.574 | 0.504 | 0.515 | 0.492 | 0.439 | 0.498 | 0.522 | 0.500 | 0.486 | 0.494 | 0.429 |
| $\mathbf{ES_{0.95}}$ | | | | | | | | | | | | | | | | | | |
| $T = 75$ | 0.428 | 0.642 | 0.408 | 0.576 | 0.365 | 0.485 | 0.494 | 0.757 | 0.507 | 0.678 | 0.484 | 0.550 | 0.504 | 0.660 | 0.501 | 0.615 | 0.491 | 0.542 |
| $T = 250$ | 0.457 | 0.704 | 0.444 | 0.598 | 0.409 | 0.472 | 0.505 | 0.831 | 0.498 | 0.700 | 0.498 | 0.551 | 0.502 | 0.702 | 0.503 | 0.632 | 0.488 | 0.522 |
| $T = 2500$ | 0.486 | 0.763 | 0.478 | 0.622 | 0.462 | 0.460 | 0.505 | 0.898 | 0.504 | 0.728 | 0.492 | 0.550 | 0.498 | 0.734 | 0.500 | 0.647 | 0.494 | 0.517 |
| $\mathbf{MVaR_{0.95}}$ | | | | | | | | | | | | | | | | | | |
| $T = 75$ | 0.448 | 0.670 | 0.437 | 0.626 | 0.410 | 0.554 | 0.494 | 0.734 | 0.507 | 0.671 | 0.484 | 0.551 | 0.504 | 0.649 | 0.501 | 0.610 | 0.491 | 0.548 |
| $T = 250$ | 0.469 | 0.749 | 0.461 | 0.672 | 0.439 | 0.560 | 0.505 | 0.815 | 0.498 | 0.696 | 0.498 | 0.554 | 0.502 | 0.697 | 0.503 | 0.630 | 0.488 | 0.526 |
| $T = 2500$ | 0.489 | 0.826 | 0.483 | 0.715 | 0.474 | 0.566 | 0.505 | 0.901 | 0.504 | 0.722 | 0.492 | 0.548 | 0.498 | 0.734 | 0.500 | 0.650 | 0.494 | 0.517 |
| $\mathbf{EVaR_{0.95}}$ | | | | | | | | | | | | | | | | | | |
| $T = 75$ | 0.459 | 0.582 | 0.450 | 0.545 | 0.424 | 0.487 | 0.494 | 0.659 | 0.507 | 0.611 | 0.484 | 0.516 | 0.504 | 0.593 | 0.501 | 0.562 | 0.491 | 0.514 |
| $T = 250$ | 0.477 | 0.633 | 0.469 | 0.565 | 0.447 | 0.476 | 0.505 | 0.735 | 0.498 | 0.635 | 0.498 | 0.516 | 0.502 | 0.633 | 0.503 | 0.580 | 0.488 | 0.495 |
| $T = 2500$ | 0.493 | 0.703 | 0.486 | 0.590 | 0.477 | 0.473 | 0.505 | 0.821 | 0.504 | 0.665 | 0.492 | 0.519 | 0.498 | 0.672 | 0.500 | 0.598 | 0.494 | 0.490 |

Table S.10: Mean severity of risk underrating for Poisson INAR(2), INARCH(1) and INARCH(2) DGPs, coherent ("Coh") vs. approximate ("Gau") forecasts, for different $(\alpha, T)$, $\alpha_2 = 0.45$ in case of of the second-order models and forecast horizon $h = 1$, computed across all simulation runs and all levels of $\mu$.

| | Poi-INAR(2) DGP | | | | | | Poi-INARCH(1) DGP | | | | | | Poi-INARCH(2) DGP | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\alpha = 0.33$ | | $\alpha = 0.55$ | | $\alpha = 0.80$ | | $\alpha = 0.33$ | | $\alpha = 0.55$ | | $\alpha = 0.80$ | | $\alpha = 0.33$ | | $\alpha = 0.55$ | | $\alpha = 0.80$ | |
| | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau |
| $\mathbf{VaR_{0.95}}$ | | | | | | | | | | | | | | | | | | |
| $T = 75$ | -1.053 | -0.639 | -1.063 | -0.702 | -1.108 | -0.849 | -1.061 | -0.660 | -1.104 | -0.804 | -1.206 | -1.221 | -1.132 | -0.855 | -1.185 | -1.025 | -1.295 | -1.423 |
| $T = 250$ | -1.005 | -0.481 | -1.005 | -0.541 | -1.012 | -0.708 | -1.005 | -0.520 | -1.005 | -0.648 | -1.037 | -1.074 | -1.021 | -0.670 | -1.031 | -0.821 | -1.074 | -1.247 |
| $T = 2500$ | -1.000 | -0.401 | -1.000 | -0.459 | -1.000 | -0.622 | -1.000 | -0.434 | -1.000 | -0.584 | -1.000 | -0.955 | -1.000 | -0.576 | -1.000 | -0.720 | -1.001 | -1.111 |
| $\mathbf{TCE_{0.95}}$ | | | | | | | | | | | | | | | | | | |
| $T = 75$ | -0.469 | -0.616 | -0.474 | -0.684 | -0.548 | -0.838 | -0.428 | -0.663 | -0.445 | -0.858 | -0.529 | -1.380 | -0.551 | -0.905 | -0.601 | -1.109 | -0.720 | -1.587 |
| $T = 250$ | -0.258 | -0.439 | -0.249 | -0.516 | -0.251 | -0.720 | -0.231 | -0.501 | -0.237 | -0.695 | -0.251 | -1.239 | -0.298 | -0.710 | -0.305 | -0.904 | -0.358 | -1.433 |
| $T = 2500$ | -0.080 | -0.346 | -0.078 | -0.426 | -0.073 | -0.640 | -0.069 | -0.401 | -0.071 | -0.614 | -0.069 | -1.118 | -0.093 | -0.608 | -0.095 | -0.802 | -0.102 | -1.311 |
| $\mathbf{ES_{0.95}}$ | | | | | | | | | | | | | | | | | | |
| $T = 75$ | -0.407 | -0.715 | -0.399 | -0.784 | -0.426 | -0.942 | -0.430 | -0.798 | -0.446 | -0.990 | -0.542 | -1.515 | -0.550 | -1.035 | -0.604 | -1.237 | -0.725 | -1.715 |
| $T = 250$ | -0.234 | -0.529 | -0.221 | -0.601 | -0.212 | -0.804 | -0.229 | -0.633 | -0.236 | -0.818 | -0.267 | -1.348 | -0.296 | -0.835 | -0.310 | -1.026 | -0.365 | -1.556 |
| $T = 2500$ | -0.078 | -0.432 | -0.074 | -0.506 | -0.070 | -0.723 | -0.072 | -0.548 | -0.073 | -0.742 | -0.082 | -1.245 | -0.092 | -0.737 | -0.095 | -0.919 | -0.106 | -1.429 |
| $\mathbf{MVaR_{0.95}}$ | | | | | | | | | | | | | | | | | | |
| $T = 75$ | -0.381 | -0.648 | -0.379 | -0.715 | -0.410 | -0.866 | -0.398 | -0.665 | -0.415 | -0.808 | -0.505 | -1.233 | -0.509 | -0.863 | -0.558 | -1.028 | -0.676 | -1.413 |
| $T = 250$ | -0.214 | -0.474 | -0.203 | -0.535 | -0.202 | -0.706 | -0.211 | -0.508 | -0.219 | -0.645 | -0.247 | -1.058 | -0.274 | -0.667 | -0.286 | -0.816 | -0.341 | -1.235 |
| $T = 2500$ | -0.070 | -0.383 | -0.066 | -0.443 | -0.063 | -0.612 | -0.066 | -0.422 | -0.068 | -0.577 | -0.075 | -0.968 | -0.085 | -0.569 | -0.087 | -0.708 | -0.099 | -1.106 |
| $\mathbf{EVaR_{0.95}}$ | | | | | | | | | | | | | | | | | | |
| $T = 75$ | -0.362 | -0.505 | -0.365 | -0.558 | -0.395 | -0.680 | -0.367 | -0.503 | -0.384 | -0.615 | -0.468 | -0.949 | -0.471 | -0.676 | -0.517 | -0.809 | -0.626 | -1.120 |
| $T = 250$ | -0.200 | -0.333 | -0.193 | -0.379 | -0.195 | -0.513 | -0.195 | -0.350 | -0.202 | -0.454 | -0.228 | -0.770 | -0.254 | -0.483 | -0.264 | -0.596 | -0.314 | -0.915 |
| $T = 2500$ | -0.065 | -0.232 | -0.061 | -0.284 | -0.059 | -0.408 | -0.061 | -0.267 | -0.063 | -0.384 | -0.069 | -0.666 | -0.079 | -0.381 | -0.081 | -0.486 | -0.090 | -0.773 |

**Note:** In analogy to Tables S.5, S.7, and S.9, for given parametrization, the frequencies of risk underrating for the coherent TCE, ES, MVaR, and EVaR forecasts of the ll-Poi-AR(1) model agree with each other, see Table S.11. This is again explained by having the one-parameter Poisson forecast distribution.

Table S.11: Frequency of risk underrating for ll-Poi-AR(1) DGP, coherent ("Coh") vs. approximate ("Gau") forecasts, for different $(\gamma_0, \gamma_1, \gamma_2)$, $\alpha = 0.55$ and $T = 250$ and forecast horizon $h = 1$, computed across all simulation runs.

| | $\gamma_0 = 0.5$ | | | | | | $\gamma_0 = 1$ | | | | | | $\gamma_0 = 2$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\gamma_1 = 0$ | | $\gamma_1 = 0.001$ | | $\gamma_1 = 0.003$ | | $\gamma_1 = 0$ | | $\gamma_1 = 0.001$ | | $\gamma_1 = 0.003$ | | $\gamma_1 = 0$ | | $\gamma_1 = 0.001$ | | $\gamma_1 = 0.003$ | |
| | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau |
| $\mathbf{VaR_{0.95}}$ | | | | | | | | | | | | | | | | | | |
| $(\gamma_2, \gamma_3) = (0, 0)$ | 0.119 | 0.818 | 0.108 | 0.780 | 0.106 | 0.908 | 0.021 | 0.549 | 0.085 | 0.763 | 0.103 | 0.894 | 0.093 | 0.647 | 0.226 | 0.814 | 0.256 | 0.960 |
| $(\gamma_2, \gamma_3) = (0.1, 0.3)$ | 0.105 | 0.652 | 0.149 | 0.697 | 0.178 | 0.831 | 0.114 | 0.609 | 0.157 | 0.692 | 0.181 | 0.821 | 0.201 | 0.587 | 0.235 | 0.677 | 0.287 | 0.822 |
| $(\gamma_2, \gamma_3) = (0.2, 0.6)$ | 0.152 | 0.579 | 0.123 | 0.548 | 0.182 | 0.646 | 0.135 | 0.492 | 0.159 | 0.539 | 0.218 | 0.584 | 0.172 | 0.421 | 0.239 | 0.460 | 0.281 | 0.511 |
| $\mathbf{TCE_{0.95}}$ | | | | | | | | | | | | | | | | | | |
| $(\gamma_2, \gamma_3) = (0, 0)$ | 0.518 | 0.694 | 0.534 | 0.724 | 0.524 | 0.880 | 0.517 | 0.480 | 0.489 | 0.694 | 0.491 | 0.876 | 0.505 | 0.555 | 0.519 | 0.772 | 0.515 | 0.957 |
| $(\gamma_2, \gamma_3) = (0.1, 0.3)$ | 0.533 | 0.557 | 0.513 | 0.619 | 0.521 | 0.797 | 0.501 | 0.524 | 0.503 | 0.637 | 0.492 | 0.779 | 0.493 | 0.513 | 0.501 | 0.621 | 0.494 | 0.809 |
| $(\gamma_2, \gamma_3) = (0.2, 0.6)$ | 0.508 | 0.473 | 0.517 | 0.504 | 0.538 | 0.600 | 0.511 | 0.438 | 0.528 | 0.488 | 0.516 | 0.548 | 0.483 | 0.363 | 0.477 | 0.425 | 0.490 | 0.501 |
| $\mathbf{ES_{0.95}}$ | | | | | | | | | | | | | | | | | | |
| $(\gamma_2, \gamma_3) = (0, 0)$ | 0.518 | 0.805 | 0.534 | 0.832 | 0.524 | 0.937 | 0.517 | 0.798 | 0.489 | 0.848 | 0.491 | 0.936 | 0.505 | 0.789 | 0.519 | 0.877 | 0.515 | 0.979 |
| $(\gamma_2, \gamma_3) = (0.1, 0.3)$ | 0.533 | 0.763 | 0.513 | 0.816 | 0.521 | 0.900 | 0.501 | 0.754 | 0.503 | 0.778 | 0.492 | 0.885 | 0.493 | 0.661 | 0.501 | 0.757 | 0.494 | 0.879 |
| $(\gamma_2, \gamma_3) = (0.2, 0.6)$ | 0.508 | 0.602 | 0.517 | 0.636 | 0.538 | 0.745 | 0.511 | 0.552 | 0.528 | 0.592 | 0.516 | 0.677 | 0.483 | 0.439 | 0.477 | 0.492 | 0.490 | 0.541 |
| $\mathbf{MVaR_{0.95}}$ | | | | | | | | | | | | | | | | | | |
| $(\gamma_2, \gamma_3) = (0, 0)$ | 0.518 | 0.828 | 0.534 | 0.870 | 0.524 | 0.933 | 0.517 | 0.808 | 0.489 | 0.839 | 0.491 | 0.929 | 0.505 | 0.770 | 0.519 | 0.849 | 0.515 | 0.965 |
| $(\gamma_2, \gamma_3) = (0.1, 0.3)$ | 0.533 | 0.811 | 0.513 | 0.832 | 0.521 | 0.906 | 0.501 | 0.765 | 0.503 | 0.780 | 0.492 | 0.866 | 0.493 | 0.649 | 0.501 | 0.725 | 0.494 | 0.846 |
| $(\gamma_2, \gamma_3) = (0.2, 0.6)$ | 0.508 | 0.673 | 0.517 | 0.677 | 0.538 | 0.743 | 0.511 | 0.564 | 0.528 | 0.604 | 0.516 | 0.652 | 0.483 | 0.430 | 0.477 | 0.475 | 0.490 | 0.520 |
| $\mathbf{EVaR_{0.95}}$ | | | | | | | | | | | | | | | | | | |
| $(\gamma_2, \gamma_3) = (0, 0)$ | 0.518 | 0.744 | 0.534 | 0.808 | 0.524 | 0.899 | 0.517 | 0.749 | 0.489 | 0.788 | 0.491 | 0.897 | 0.505 | 0.725 | 0.519 | 0.805 | 0.515 | 0.957 |
| $(\gamma_2, \gamma_3) = (0.1, 0.3)$ | 0.533 | 0.701 | 0.513 | 0.744 | 0.521 | 0.853 | 0.501 | 0.676 | 0.503 | 0.717 | 0.492 | 0.808 | 0.493 | 0.598 | 0.501 | 0.677 | 0.494 | 0.787 |
| $(\gamma_2, \gamma_3) = (0.2, 0.6)$ | 0.508 | 0.536 | 0.517 | 0.577 | 0.538 | 0.655 | 0.511 | 0.490 | 0.528 | 0.556 | 0.516 | 0.580 | 0.483 | 0.424 | 0.477 | 0.461 | 0.490 | 0.491 |

Table S.12: Frequency of risk underrating for ll-Poi-AR(1) DGP, coherent ("Coh") vs. approximate ("Gau") forecasts, for different $(\gamma_0, \gamma_1, \gamma_2)$, $\alpha = 0.55$ and $T = 250$ and forecast horizon $h = 1$, computed across all simulation runs.

| | $\gamma_0 = 0.5$ | | | | | | $\gamma_0 = 1$ | | | | | | $\gamma_0 = 2$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\gamma_1 = 0$ | | $\gamma_1 = 0.001$ | | $\gamma_1 = 0.003$ | | $\gamma_1 = 0$ | | $\gamma_1 = 0.001$ | | $\gamma_1 = 0.003$ | | $\gamma_1 = 0$ | | $\gamma_1 = 0.001$ | | $\gamma_1 = 0.003$ | |
| | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau |
| $\mathbf{VaR_{0.95}}$ | | | | | | | | | | | | | | | | | | |
| $(\gamma_2; \gamma_3) = (0; 0)$ | -1.000 | -0.484 | -1.009 | -0.655 | -1.009 | -0.886 | -1.000 | -0.465 | -1.000 | -0.634 | -1.000 | -1.080 | -1.011 | -0.575 | -1.022 | -0.921 | -1.055 | -1.711 |
| $(\gamma_2; \gamma_3) = (1; 1)$ | -1.000 | -0.514 | -1.000 | -0.688 | -1.011 | -1.066 | -1.000 | -0.618 | -1.013 | -0.804 | -1.028 | -1.229 | -1.020 | -0.844 | -1.077 | -1.126 | -1.202 | -2.156 |
| $(\gamma_2; \gamma_3) = (2; 2)$ | -1.000 | -0.727 | -1.000 | -0.879 | -1.027 | -1.463 | -1.030 | -0.881 | -1.038 | -1.173 | -1.046 | -1.906 | -1.064 | -1.452 | -1.105 | -2.118 | -1.217 | -3.614 |
| $\mathbf{TCE_{0.95}}$ | | | | | | | | | | | | | | | | | | |
| $(\gamma_2, \gamma_3) = (0, 0)$ | -0.235 | -0.454 | -0.221 | -0.645 | -0.228 | -0.934 | -0.067 | -0.470 | -0.197 | -0.651 | -0.239 | -1.160 | -0.204 | -0.602 | -0.462 | -0.960 | -0.547 | -1.862 |
| $(\gamma_2, \gamma_3) = (0.1, 0.3)$ | -0.218 | -0.521 | -0.311 | -0.720 | -0.369 | -1.138 | -0.252 | -0.654 | -0.339 | -0.840 | -0.410 | -1.362 | -0.433 | -0.927 | -0.530 | -1.243 | -0.728 | -2.377 |
| $(\gamma_2, \gamma_3) = (0.2, 0.6)$ | -0.312 | -0.827 | -0.266 | -0.964 | -0.373 | -1.668 | -0.290 | -0.984 | -0.335 | -1.338 | -0.469 | -2.181 | -0.399 | -1.784 | -0.580 | -2.468 | -0.729 | -4.042 |
| $\mathbf{ES_{0.95}}$ | | | | | | | | | | | | | | | | | | |
| $(\gamma_2, \gamma_3) = (0, 0)$ | -0.139 | -0.620 | -0.208 | -0.821 | -0.266 | -1.286 | -0.192 | -0.656 | -0.241 | -0.905 | -0.314 | -1.500 | -0.239 | -0.800 | -0.425 | -1.197 | -0.541 | -2.222 |
| $(\gamma_2, \gamma_3) = (0.1, 0.3)$ | -0.229 | -0.666 | -0.287 | -0.839 | -0.356 | -1.341 | -0.280 | -0.759 | -0.336 | -1.004 | -0.449 | -1.566 | -0.394 | -1.038 | -0.522 | -1.355 | -0.720 | -2.535 |
| $(\gamma_2, \gamma_3) = (0.2, 0.6)$ | -0.246 | -0.872 | -0.308 | -1.097 | -0.366 | -1.659 | -0.274 | -1.073 | -0.329 | -1.399 | -0.467 | -2.078 | -0.387 | -1.810 | -0.567 | -2.470 | -0.735 | -4.149 |
| $\mathbf{MVaR_{0.95}}$ | | | | | | | | | | | | | | | | | | |
| $(\gamma_2, \gamma_3) = (0, 0)$ | -0.125 | -0.538 | -0.215 | -0.684 | -0.236 | -1.048 | -0.161 | -0.554 | -0.215 | -0.737 | -0.297 | -1.209 | -0.221 | -0.619 | -0.400 | -0.951 | -0.517 | -1.788 |
| $(\gamma_2, \gamma_3) = (0.1, 0.3)$ | -0.201 | -0.563 | -0.252 | -0.707 | -0.322 | -1.093 | -0.250 | -0.620 | -0.306 | -0.819 | -0.422 | -1.287 | -0.370 | -0.822 | -0.492 | -1.111 | -0.687 | -2.121 |
| $(\gamma_2, \gamma_3) = (0.2, 0.6)$ | -0.216 | -0.670 | -0.276 | -0.866 | -0.334 | -1.370 | -0.242 | -0.851 | -0.299 | -1.114 | -0.429 | -1.766 | -0.368 | -1.466 | -0.536 | -2.089 | -0.705 | -3.637 |
| $\mathbf{EVaR_{0.95}}$ | | | | | | | | | | | | | | | | | | |
| $(\gamma_2, \gamma_3) = (0, 0)$ | -0.126 | -0.319 | -0.188 | -0.451 | -0.220 | -0.729 | -0.143 | -0.360 | -0.201 | -0.498 | -0.271 | -0.871 | -0.208 | -0.436 | -0.373 | -0.702 | -0.491 | -1.332 |
| $(\gamma_2, \gamma_3) = (0.1, 0.3)$ | -0.180 | -0.371 | -0.227 | -0.487 | -0.293 | -0.791 | -0.229 | -0.438 | -0.278 | -0.593 | -0.385 | -0.976 | -0.344 | -0.630 | -0.461 | -0.870 | -0.652 | -1.742 |
| $(\gamma_2, \gamma_3) = (0.2, 0.6)$ | -0.191 | -0.513 | -0.244 | -0.666 | -0.312 | -1.116 | -0.222 | -0.647 | -0.280 | -0.849 | -0.395 | -1.490 | -0.342 | -1.112 | -0.503 | -1.682 | -0.669 | -3.131 |

## Section 4.3 DGPs for Bounded Counts

Table S.13: Frequency of risk underrating for BinAR(1) and BinARCH(1) DGPs, coherent (“Coh”) vs. approximate (“Gau”) forecasts, for $\pi = 0.15$ and different $T$ and forecast horizon $h = 1$, computed across all simulation runs and $n = 10, \ldots, 130$.

| | BinAR(1) DGP | | | | | | BinARCH(1) DGP | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\alpha = 0.33$ | | $\alpha = 0.55$ | | $\alpha = 0.80$ | | $\alpha = 0.33$ | | $\alpha = 0.55$ | | $\alpha = 0.80$ | |
| | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau |
| **$\mathrm{VaR}_{0.95}$** | | | | | | | | | | | | |
| $T = 75$ | 0.184 | 0.579 | 0.156 | 0.553 | 0.091 | 0.478 | 0.205 | 0.600 | 0.204 | 0.573 | 0.219 | 0.528 |
| $T = 250$ | 0.106 | 0.607 | 0.097 | 0.569 | 0.076 | 0.494 | 0.118 | 0.632 | 0.119 | 0.601 | 0.122 | 0.547 |
| $T = 2500$ | 0.032 | 0.627 | 0.033 | 0.582 | 0.033 | 0.506 | 0.039 | 0.660 | 0.038 | 0.623 | 0.036 | 0.566 |
| **$\mathrm{TCE}_{0.95}$** | | | | | | | | | | | | |
| $T = 75$ | 0.469 | 0.446 | 0.401 | 0.400 | 0.278 | 0.304 | 0.503 | 0.485 | 0.503 | 0.472 | 0.504 | 0.459 |
| $T = 250$ | 0.482 | 0.413 | 0.441 | 0.351 | 0.370 | 0.257 | 0.503 | 0.468 | 0.503 | 0.462 | 0.497 | 0.460 |
| $T = 2500$ | 0.492 | 0.382 | 0.479 | 0.312 | 0.458 | 0.227 | 0.503 | 0.450 | 0.503 | 0.456 | 0.500 | 0.468 |
| **$\mathrm{ES}_{0.95}$** | | | | | | | | | | | | |
| $T = 75$ | 0.489 | 0.669 | 0.434 | 0.628 | 0.301 | 0.556 | 0.503 | 0.685 | 0.503 | 0.644 | 0.504 | 0.572 |
| $T = 250$ | 0.494 | 0.771 | 0.463 | 0.709 | 0.386 | 0.633 | 0.503 | 0.774 | 0.503 | 0.704 | 0.497 | 0.602 |
| $T = 2500$ | 0.498 | 0.936 | 0.488 | 0.852 | 0.464 | 0.769 | 0.503 | 0.893 | 0.503 | 0.756 | 0.500 | 0.624 |
| **$\mathrm{MVaR}_{0.95}$** | | | | | | | | | | | | |
| $T = 75$ | 0.493 | 0.683 | 0.449 | 0.670 | 0.326 | 0.664 | 0.503 | 0.685 | 0.503 | 0.647 | 0.504 | 0.577 |
| $T = 250$ | 0.496 | 0.786 | 0.471 | 0.765 | 0.400 | 0.781 | 0.503 | 0.773 | 0.503 | 0.712 | 0.497 | 0.609 |
| $T = 2500$ | 0.499 | 0.944 | 0.490 | 0.909 | 0.468 | 0.925 | 0.503 | 0.903 | 0.503 | 0.768 | 0.500 | 0.636 |
| **$\mathrm{EVaR}_{0.95}$** | | | | | | | | | | | | |
| $T = 75$ | 0.495 | 0.602 | 0.462 | 0.578 | 0.359 | 0.529 | 0.503 | 0.613 | 0.503 | 0.587 | 0.504 | 0.537 |
| $T = 250$ | 0.497 | 0.674 | 0.479 | 0.634 | 0.419 | 0.578 | 0.503 | 0.680 | 0.503 | 0.636 | 0.497 | 0.564 |
| $T = 2500$ | 0.499 | 0.851 | 0.493 | 0.771 | 0.475 | 0.688 | 0.503 | 0.821 | 0.503 | 0.699 | 0.500 | 0.596 |

Table S.14: Mean severity of risk underrating for BinAR(1) and BinARCH(1) DGPs, coherent (“Coh”) vs. approximate (“Gau”) forecasts, for $\pi = 0.15$ and different $T$ and forecast horizon $h = 1$, computed across all simulation runs and $n = 10, \ldots, 130$.

| | BinAR(1) DGP | | | | | | BinARCH(1) DGP | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\alpha = 0.33$ | | $\alpha = 0.55$ | | $\alpha = 0.80$ | | $\alpha = 0.33$ | | $\alpha = 0.55$ | | $\alpha = 0.80$ | |
| | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau |
| **$\mathrm{VaR}_{0.95}$** | | | | | | | | | | | | |
| $T = 75$ | -1.059 | -0.609 | -1.043 | -0.558 | -1.015 | -0.444 | -1.078 | -0.670 | -1.103 | -0.748 | -1.180 | -0.988 |
| $T = 250$ | -1.006 | -0.430 | -1.004 | -0.398 | -1.001 | -0.326 | -1.010 | -0.476 | -1.013 | -0.548 | -1.028 | -0.759 |
| $T = 2500$ | -1.000 | -0.339 | -1.000 | -0.320 | -1.000 | -0.275 | -1.000 | -0.375 | -1.000 | -0.447 | -1.000 | -0.659 |
| **$\mathrm{TCE}_{0.95}$** | | | | | | | | | | | | |
| $T = 75$ | -0.440 | -0.571 | -0.438 | -0.509 | -0.374 | -0.382 | -0.454 | -0.645 | -0.463 | -0.745 | -0.529 | -1.019 |
| $T = 250$ | -0.237 | -0.353 | -0.245 | -0.315 | -0.238 | -0.234 | -0.245 | -0.413 | -0.249 | -0.513 | -0.261 | -0.773 |
| $T = 2500$ | -0.071 | -0.218 | -0.078 | -0.199 | -0.085 | -0.148 | -0.079 | -0.280 | -0.078 | -0.391 | -0.076 | -0.661 |
| **$\mathrm{ES}_{0.95}$** | | | | | | | | | | | | |
| $T = 75$ | -0.425 | -0.671 | -0.396 | -0.593 | -0.340 | -0.432 | -0.450 | -0.757 | -0.466 | -0.856 | -0.532 | -1.148 |
| $T = 250$ | -0.235 | -0.441 | -0.231 | -0.383 | -0.225 | -0.265 | -0.245 | -0.522 | -0.249 | -0.626 | -0.264 | -0.912 |
| $T = 2500$ | -0.076 | -0.302 | -0.077 | -0.252 | -0.082 | -0.163 | -0.077 | -0.389 | -0.077 | -0.510 | -0.079 | -0.815 |
| **$\mathrm{MVaR}_{0.95}$** | | | | | | | | | | | | |
| $T = 75$ | -0.402 | -0.604 | -0.360 | -0.553 | -0.275 | -0.446 | -0.433 | -0.663 | -0.448 | -0.745 | -0.513 | -0.995 |
| $T = 250$ | -0.221 | -0.404 | -0.206 | -0.372 | -0.179 | -0.308 | -0.236 | -0.454 | -0.239 | -0.532 | -0.254 | -0.765 |
| $T = 2500$ | -0.071 | -0.283 | -0.068 | -0.264 | -0.065 | -0.235 | -0.074 | -0.329 | -0.074 | -0.423 | -0.075 | -0.666 |
| **$\mathrm{EVaR}_{0.95}$** | | | | | | | | | | | | |
| $T = 75$ | -0.379 | -0.478 | -0.336 | -0.434 | -0.242 | -0.334 | -0.412 | -0.527 | -0.427 | -0.588 | -0.491 | -0.789 |
| $T = 250$ | -0.208 | -0.289 | -0.188 | -0.260 | -0.150 | -0.190 | -0.225 | -0.329 | -0.228 | -0.385 | -0.243 | -0.552 |
| $T = 2500$ | -0.066 | -0.157 | -0.061 | -0.140 | -0.054 | -0.098 | -0.070 | -0.197 | -0.071 | -0.270 | -0.072 | -0.442 |

**Note:** In analogy to Tables S.5, S.7, and S.9, for given $(\alpha, T)$, the frequencies of risk underrating for the coherent TCE, ES, MVaR, and EVaR forecasts of the BinARCH(1) model nearly agree with each other, see Tables S.13 and S.15. This is again explained by having a one-parameter forecast distribution, now the binomial distribution. However, for $\alpha = 0.80$, there are sometimes slight deviations. These occur if some of the simulated risk forecasts became equal to the upper bound $n$ of the range, because then, there might be exact matches instead of an underrating.

Table S.15: Frequency of risk underrating for BinAR(1) and BinARCH(1) DGPs, coherent ("Coh") vs. approximate ("Gau") forecasts, for $\pi = 0.45$ and different $T$ and forecast horizon $h = 1$, computed across all simulation runs and $n = 10, \dots, 130$.

| | BinAR(1) DGP | | | | | | BinARCH(1) DGP | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\alpha = 0.33$ | | $\alpha = 0.55$ | | $\alpha = 0.80$ | | $\alpha = 0.33$ | | $\alpha = 0.55$ | | $\alpha = 0.80$ | |
| | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau |
| $\mathbf{VaR_{0.95}}$ | | | | | | | | | | | | |
| $T = 75$ | 0.205 | 0.498 | 0.185 | 0.495 | 0.109 | 0.448 | 0.228 | 0.515 | 0.227 | 0.504 | 0.234 | 0.463 |
| $T = 250$ | 0.120 | 0.488 | 0.120 | 0.496 | 0.096 | 0.459 | 0.139 | 0.518 | 0.136 | 0.505 | 0.135 | 0.489 |
| $T = 2500$ | 0.031 | 0.476 | 0.044 | 0.495 | 0.041 | 0.473 | 0.046 | 0.521 | 0.044 | 0.513 | 0.042 | 0.508 |
| $\mathbf{TCE_{0.95}}$ | | | | | | | | | | | | |
| $T = 75$ | 0.464 | 0.332 | 0.400 | 0.319 | 0.268 | 0.257 | 0.499 | 0.350 | 0.500 | 0.342 | 0.500 | 0.313 |
| $T = 250$ | 0.480 | 0.234 | 0.440 | 0.223 | 0.367 | 0.183 | 0.501 | 0.260 | 0.500 | 0.251 | 0.499 | 0.248 |
| $T = 2500$ | 0.493 | 0.093 | 0.480 | 0.106 | 0.455 | 0.110 | 0.500 | 0.120 | 0.501 | 0.120 | 0.499 | 0.143 |
| $\mathbf{ES_{0.95}}$ | | | | | | | | | | | | |
| $T = 75$ | 0.489 | 0.511 | 0.431 | 0.498 | 0.287 | 0.461 | 0.499 | 0.510 | 0.500 | 0.501 | 0.500 | 0.460 |
| $T = 250$ | 0.495 | 0.509 | 0.462 | 0.496 | 0.380 | 0.481 | 0.501 | 0.513 | 0.500 | 0.506 | 0.499 | 0.495 |
| $T = 2500$ | 0.499 | 0.512 | 0.488 | 0.504 | 0.461 | 0.528 | 0.500 | 0.524 | 0.501 | 0.537 | 0.499 | 0.580 |
| $\mathbf{MVaR_{0.95}}$ | | | | | | | | | | | | |
| $T = 75$ | 0.491 | 0.573 | 0.445 | 0.571 | 0.310 | 0.572 | 0.499 | 0.568 | 0.500 | 0.559 | 0.500 | 0.518 |
| $T = 250$ | 0.497 | 0.620 | 0.470 | 0.626 | 0.393 | 0.667 | 0.501 | 0.616 | 0.500 | 0.606 | 0.500 | 0.588 |
| $T = 2500$ | 0.499 | 0.783 | 0.490 | 0.803 | 0.466 | 0.883 | 0.500 | 0.775 | 0.501 | 0.768 | 0.499 | 0.735 |
| $\mathbf{EVaR_{0.95}}$ | | | | | | | | | | | | |
| $T = 75$ | 0.494 | 0.518 | 0.461 | 0.509 | 0.349 | 0.480 | 0.499 | 0.516 | 0.500 | 0.511 | 0.501 | 0.481 |
| $T = 250$ | 0.498 | 0.525 | 0.479 | 0.515 | 0.415 | 0.497 | 0.501 | 0.527 | 0.500 | 0.523 | 0.500 | 0.515 |
| $T = 2500$ | 0.500 | 0.563 | 0.493 | 0.555 | 0.471 | 0.549 | 0.500 | 0.571 | 0.501 | 0.579 | 0.500 | 0.599 |

Table S.16: Mean severity of risk underrating for BinAR(1) and BinARCH(1) DGPs, coherent ("Coh") vs. approximate ("Gau") forecasts, for $\pi = 0.45$ and different $T$ and forecast horizon $h = 1$, computed across all simulation runs and $n = 10, \dots, 130$.

| | BinAR(1) DGP | | | | | | BinARCH(1) DGP | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\alpha = 0.33$ | | $\alpha = 0.55$ | | $\alpha = 0.80$ | | $\alpha = 0.33$ | | $\alpha = 0.55$ | | $\alpha = 0.80$ | |
| | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau | Coh | Gau |
| $\mathbf{VaR_{0.95}}$ | | | | | | | | | | | | |
| $T = 75$ | -1.114 | -0.675 | -1.101 | -0.624 | -1.058 | -0.487 | -1.135 | -0.729 | -1.146 | -0.732 | -1.210 | -0.762 |
| $T = 250$ | -1.019 | -0.416 | -1.020 | -0.390 | -1.011 | -0.319 | -1.022 | -0.452 | -1.023 | -0.450 | -1.030 | -0.453 |
| $T = 2500$ | -1.000 | -0.259 | -1.000 | -0.264 | -1.000 | -0.246 | -1.000 | -0.282 | -1.000 | -0.280 | -1.000 | -0.293 |
| $\mathbf{TCE_{0.95}}$ | | | | | | | | | | | | |
| $T = 75$ | -0.512 | -0.638 | -0.539 | -0.574 | -0.482 | -0.429 | -0.516 | -0.694 | -0.519 | -0.697 | -0.562 | -0.726 |
| $T = 250$ | -0.270 | -0.334 | -0.305 | -0.305 | -0.307 | -0.223 | -0.282 | -0.372 | -0.277 | -0.370 | -0.277 | -0.368 |
| $T = 2500$ | -0.070 | -0.104 | -0.101 | -0.101 | -0.107 | -0.087 | -0.090 | -0.123 | -0.087 | -0.120 | -0.083 | -0.132 |
| $\mathbf{ES_{0.95}}$ | | | | | | | | | | | | |
| $T = 75$ | -0.505 | -0.722 | -0.488 | -0.641 | -0.438 | -0.473 | -0.508 | -0.763 | -0.518 | -0.768 | -0.566 | -0.789 |
| $T = 250$ | -0.282 | -0.396 | -0.284 | -0.348 | -0.294 | -0.253 | -0.277 | -0.420 | -0.278 | -0.417 | -0.282 | -0.423 |
| $T = 2500$ | -0.090 | -0.130 | -0.095 | -0.115 | -0.106 | -0.092 | -0.086 | -0.138 | -0.086 | -0.140 | -0.086 | -0.166 |
| $\mathbf{MVaR_{0.95}}$ | | | | | | | | | | | | |
| $T = 75$ | -0.495 | -0.670 | -0.455 | -0.603 | -0.364 | -0.466 | -0.510 | -0.704 | -0.521 | -0.711 | -0.571 | -0.741 |
| $T = 250$ | -0.275 | -0.382 | -0.261 | -0.343 | -0.241 | -0.274 | -0.278 | -0.402 | -0.279 | -0.402 | -0.284 | -0.413 |
| $T = 2500$ | -0.087 | -0.162 | -0.086 | -0.154 | -0.087 | -0.152 | -0.087 | -0.167 | -0.087 | -0.172 | -0.086 | -0.203 |
| $\mathbf{EVaR_{0.95}}$ | | | | | | | | | | | | |
| $T = 75$ | -0.483 | -0.568 | -0.432 | -0.512 | -0.316 | -0.391 | -0.508 | -0.602 | -0.519 | -0.610 | -0.570 | -0.647 |
| $T = 250$ | -0.267 | -0.313 | -0.241 | -0.276 | -0.197 | -0.202 | -0.277 | -0.331 | -0.278 | -0.330 | -0.284 | -0.335 |
| $T = 2500$ | -0.084 | -0.102 | -0.079 | -0.090 | -0.070 | -0.068 | -0.086 | -0.108 | -0.087 | -0.109 | -0.086 | -0.123 |