Evaluating Approximate Point Forecasting of Count Processes

A. Homburg, C.H. Weiß, G. Frahm
Department of Mathematics & Statistics, Helmut Schmidt University, Hamburg

L.C. Alwan
Sheldon B. Lubar School of Business, University of Wisconsin-Milwaukee

R. Göb
Department of Statistics, University of Würzburg
Point Forecasting of Count Processes

Introduction
Let \((X_t)_{t \in \mathbb{Z}}\) be count process,
let \(x_1, \ldots, x_T\) be count time series thereof,
i.e., \(x_t\) non-negative integer values from \(\mathbb{N}_0 = \{0, 1, \ldots\}\).

After fitting model to \(x_1, \ldots, x_T\), aim to
predict outcome of \(X_{T+h}\) for some forecast horizon \(h \geq 1\)
by computing point forecast \(\hat{x}_{T+h}\).

Since \(X_{T+h}\) will take count value,
also point forecast \(\hat{x}_{T+h}\) should be count value
\(\rightarrow\) coherent forecasting (Freeland & McCabe, 2004).
Coherent forecasting achieved by computing $h$-step-ahead conditional distribution of $X_{T+h}$ (given $x_T, \ldots, x_1$) of actual count process model, and by deriving integer-valued quantity as forecast value.

- Coherent **central** point forecast:
  commonly **conditional median**.

- Coherent **non-central** point forecast:
  **conditional (upper) quantile** (Value at Risk, VaR),

\[
\text{VaR}_\rho = \min \{ x \in \mathbb{N}_0 \mid P(X \leq x) \geq \rho \},
\]

especially risk analysis (loss distribution), see Göb (2011).

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However, practitioners often apply Gaussian (ARIMA) approximations, and compute conditional median/quantiles thereof (with additional ceiling to ensure count values).

How well do these approximate forecasts perform compared to coherent forecasting techniques?

We analyzed approximation quality of both central and non-central point forecasts in comprehensive way by considering different types of DGP, various model parametrizations as well as different experimental designs.

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Full results in Homburg et al. (2019), Evaluating approximate point forecasting of count processes, *Econometrics* 7, 30. (open access)
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Performance Evaluation
Performance of approximate forecasts evaluated based on selected inaccuracy measures, relative to forecasts computed from true count model.

**Inaccuracy of central point forecasts** (i.e., median):
Mean Absolute Error (MAE),

$$\text{MAE} = E\left[ |X_{T+h} - \hat{x}_{T+h}| \mid x_T, \ldots, x_1 \right],$$

corresponding relative measure

$$\text{RMAE} = \frac{\text{MAE}_f}{\text{MAE}_t}. $$
Inaccuracy of non-central point forecasts (i.e., VaR$_\rho$): one may use asymmetric “lin-lin” loss function (Gneiting, 2011),

$$\rho |X_{T+h} - \hat{x}_{T+h}| \cdot 1\{X_{T+h} > \hat{x}_{T+h}\} + (1-\rho) |X_{T+h} - \hat{x}_{T+h}| \cdot 1\{X_{T+h} < \hat{x}_{T+h}\}.$$  

But in risk context, only penalize exceedances (Lopez, 1998). Mean Excess Loss (MEL),

$$\text{MEL} = E[(X_{T+h} - \hat{x}_{T+h}) 1\{X_{T+h} > \hat{x}_{T+h}\} \mid x_T, \ldots, x_1],$$

corresponding relative measure

$$\text{RMEL} = \frac{\text{MEL}_f}{\text{MEL}_t}.$$
Performance Analysis: Coherent vs. Approximate Point Forecasting

Selected Results
In what follows, time series simulated according to certain count time series model, e.g., Poisson \( \text{INAR}(1) \) process

\[
X_t = \alpha \circ X_{t-1} + \epsilon_t \quad \text{with} \quad \epsilon_t \sim \text{Poi}(\mu_X (1 - \alpha)).
\]

Then either Poi-INAR\((1)\) fitted to data and coherent forecasts, or Gaussian AR\((1)\) fitted and ceiled forecasts (both \( h = 1 \)), given \( x_T \) of respective simulation run.

Analogous procedure for further types of DGP.

Homburg et al. (2019) also provide results regarding

- pure approximation error (no estimation),
- effect of \( x_T \) or \( h \), mode point forecasts, etc.
Coherent vs. Approximate Point Forecasting

Mean of simulated RMAE values for median forecast (left) and RMEL values for 95%-quantile forecast (right), Poi-INAR(1) with $\alpha = 0.55$ and $T = 250$.

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% of RMEL values $= 1$ (central area), $< 1$ (lower area) and $> 1$ (upper area) for different $T$. Grey area: Poi-INAR(1) forecasts, areas separated by black line: Gaussian AR(1) forecasts.
Mean of simulated RMEL values for NB-INAR(1) (left) and ZIP-INAR(1) (right), with $\alpha = 0.55$, $T = 250$, and $\sigma_X^2/\mu_X = 2.4$. 

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% of RMEL values = 1, < 1 and > 1 for NB-INAR(1) (left) and ZIP-INAR(1) (right). Grey area: INAR(1) forecasts, areas separated by black line: Gaussian AR(1) forecasts.
Mean RMAE (left) and RMEL (right) for DGP Poi-INAR(2) vs. Poi-INARCH(2), with $\alpha = 0.55$, $\alpha_2 = 0.45$ and $T = 250$; solid lines smoothed values of Poi-INAR(1) or Poi-INARCH(1) case.
Coherent vs. Approximate Point Forecasting

Mean RMEL (left) and % of RMEL values = 1, < 1 and > 1 (right) for BinAR(1) with $\pi = 0.45$, $\alpha = 0.55$, and $T = 250$. Solid lines correspond to smoothed values of Poi-INAR(1) case.

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Conclusions

- Coherent point forecasting based on count model: estimation error nearly no effect on median forecasts; RMEL performance balanced and improves with increasing $T$.
- Point forecasting via Gaussian ARMA approximation: RMAE and RMEL performance considerably worse, strongly biased RMEL performance, severely affected by overdispersion or zero inflation, or by bounded counts.
- Practice of discretizing Gaussian ARIMA forecasts for count time series strongly discouraged!
- **Ongoing research:** construction and evaluation of (approximate) prediction intervals for count time series.

Christian H. Weiβ — Helmut Schmidt University, Hamburg
Thank You
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Christian H. Weiß
Department of Mathematics & Statistics
Helmut Schmidt University, Hamburg
weissc@hsu-hh.de


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