# Evaluating Approximate Point Forecasting of Count Processes

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# Point Forecasting of Count Processes

## Introduction

# Point Forecasting of Count Processes



Let $(X_t)_{t\in\mathbb{Z}}$ be count process,
let $x_1,\ldots,x_T$ be count time series thereof,
i. e., $x_t$ non-negative integer values from $\mathbb{N}_0 = \{0,1,\ldots\}$.

After fitting model to $x_1,\ldots,x_T$, aim to
predict outcome of $X_{T+h}$ for some forecast horizon $h \geq 1$
by computing point forecast $\widehat{x}_{T+h}$.

Since $X_{T+h}$ will take count value,
also point forecast $\widehat{x}_{T+h}$ should be count value
$\rightarrow$ **coherent forecasting** (Freeland & McCabe, 2004).

## Point Forecasting of Count Processes



Coherent forecasting achieved by computing $h$-step-ahead conditional distribution of $X_{T+h}$ (given $x_T, \ldots, x_1$) of actual count process model, and by deriving integer-valued quantity as forecast value.

- Coherent **central** point forecast:
  commonly **conditional median**.
- Coherent **non-central** point forecast:
  **conditional (upper) quantile** (Value at Risk, VaR),

$$\text{VaR}_\rho \;=\; \min\big\{x \in \mathbb{N}_0 \mid P(X \leq x) \geq \rho\big\},$$

  especially risk analysis (loss distribution), see Göb (2011).

However,

practioners often apply **Gaussian (ARIMA) approximations**, and compute conditional median/quantiles thereof (with additional ceiling to ensure count values).

**How well do these approximate forecasts perform compared to coherent forecasting techniques?**

We analyzed approximation quality of both central and non-central point forecasts in comprehensive way by considering different types of DGP, various model parametrizations as well as different experimental designs.

**Full results** in Homburg et al. (2019),

Evaluating approximate point forecasting of count processes,

*Econometrics* **7**, 30. (open access)

Marginal distribution

*Overdispersion:* NB-INAR(1)

*Zero inflation:* ZIP-INAR(1)

(Section 4.1)

*Bounded counts:*

BinAR(1) BinARCH(1)

(Section 4.4)

Poi-INAR(1)
(Section 3)

*Poisson counts (unbounded, equidispersed) with AR(1)-like autocorrelation structure*

Dependence structure

*Higher-order autoregression:*

Poi-INAR($p$)
(Section 4.2)

Poi-INARCH($p$)
(Section 4.3)

*Seasonality and trend:*

II-Poi-AR(1)
(Section 4.5)

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# Point Forecasting of Count Processes

## Performance Evaluation

Performance of approximate forecasts

evaluated based on selected inaccuracy measures,

relative to forecasts computed from true count model.

**Inaccuracy of central point forecasts** (i. e., median):

Mean Absolute Error (MAE),

$$\mathsf{MAE} \;=\; E\big[|X_{T+h} - \widehat{x}_{T+h}| \;\big|\; x_T, \ldots, x_1\big],$$

corresponding relative measure

$$\mathsf{RMAE} \;=\; \frac{\mathsf{MAE_f}}{\mathsf{MAE_t}}.$$

**Inaccuracy of non-central point forecasts** (i. e., $\mathsf{VaR}_\rho$):

one may use asymmetric “lin-lin” loss function (Gneiting, 2011),

$$\rho\,|X_{T+h}-\widehat{x}_{T+h}|\cdot\mathbb{1}_{\{X_{T+h}>\widehat{x}_{T+h}\}}+(1-\rho)\,|X_{T+h}-\widehat{x}_{T+h}|\cdot\mathbb{1}_{\{X_{T+h}<\widehat{x}_{T+h}\}}.$$

But in risk context, only penalize exceedances (Lopez, 1998).

Mean Excess Loss (MEL),

$$\mathsf{MEL} \;=\; E\Big[(X_{T+h}-\widehat{x}_{T+h})\,\mathbb{1}_{\{X_{T+h}>\widehat{x}_{T+h}\}}\;\Big|\;x_T,\ldots,x_1\Big],$$

corresponding relative measure

$$\mathsf{RMEL} \;=\; \frac{\mathsf{MEL_f}}{\mathsf{MEL_t}}.$$

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# Performance Analysis: Coherent vs. Approximate Point Forecasting

Selected Results

In what follows, time series simulated according to certain count time series model, e. g., Poisson INAR(1) process

$$X_t \;=\; \alpha \circ X_{t-1} + \epsilon_t \qquad \text{with } \epsilon_t \sim \text{Poi}\big(\mu_X\,(1-\alpha)\big).$$

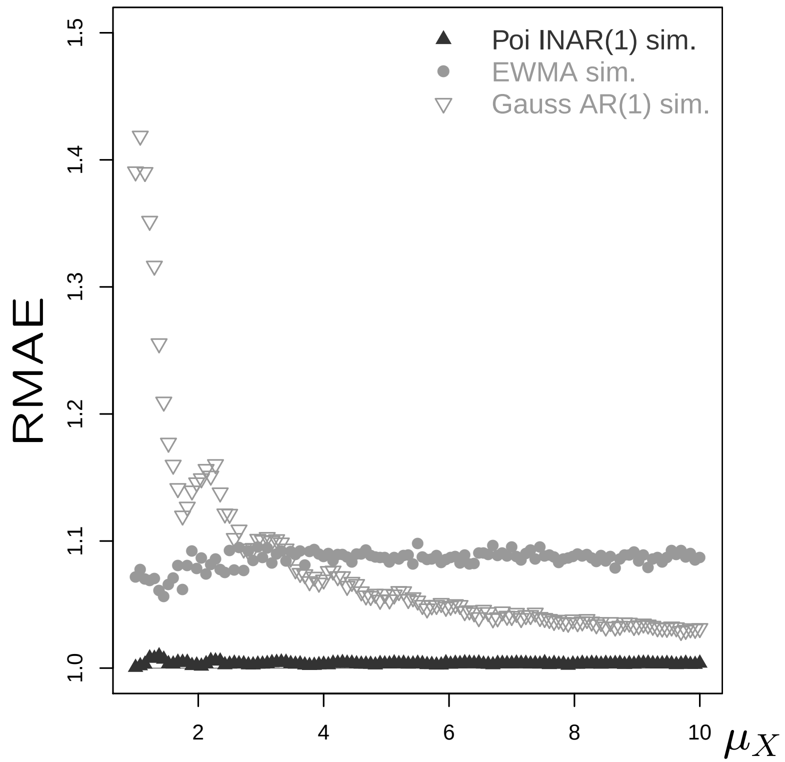
Then either Poi-INAR(1) fitted to data and coherent forecasts, or Gaussian AR(1) fitted and ceiled forecasts (both $h=1$), given $x_T$ of respective simulation run.

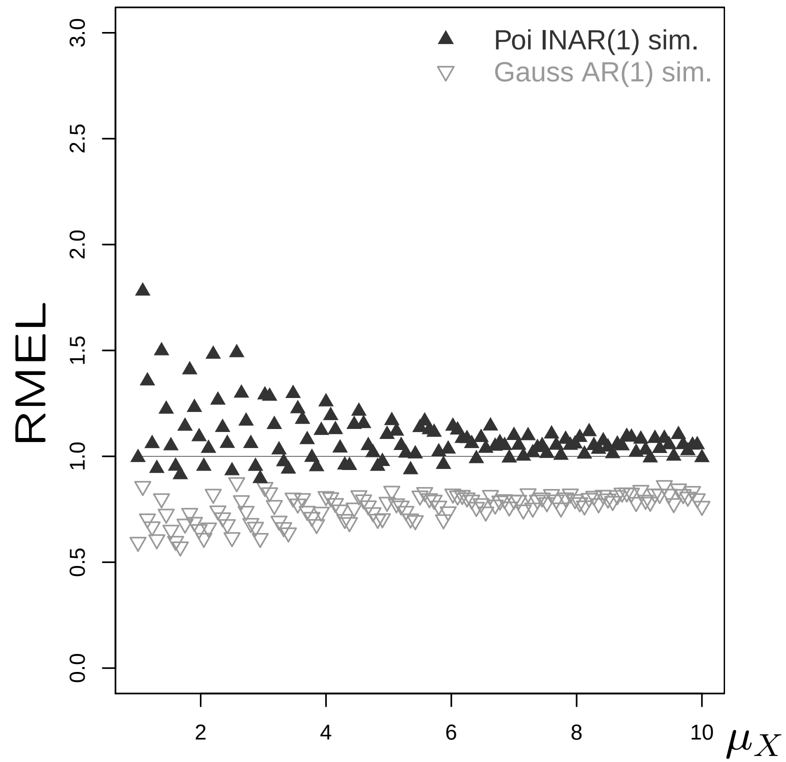
Analogous procedure for further types of DGP.

Homburg et al. (2019) also provide results regarding

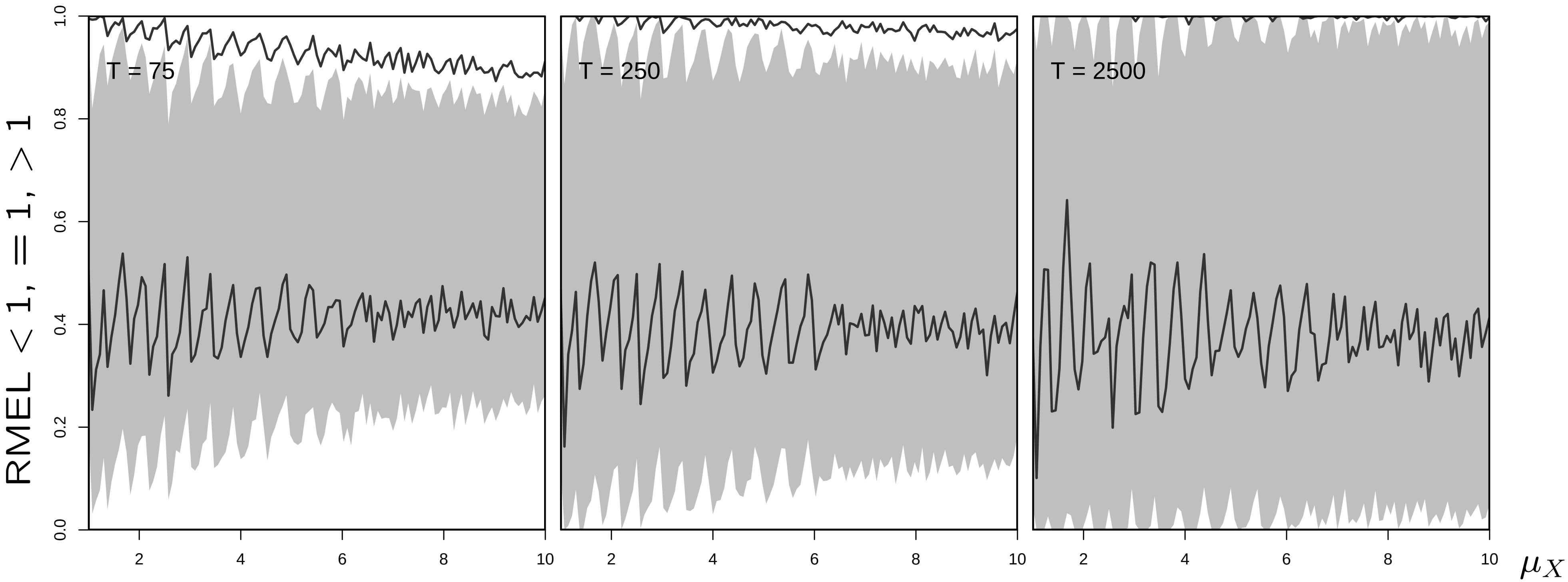
- pure approximation error (no estimation),
- effect of $x_T$ or $h$, mode point forecasts, etc.

Mean of simulated RMAE values for median forecast (left) and RMEL values for 95 %-quantile forecast (right), Poi-INAR(1) with $\alpha = 0.55$ and $T = 250$.

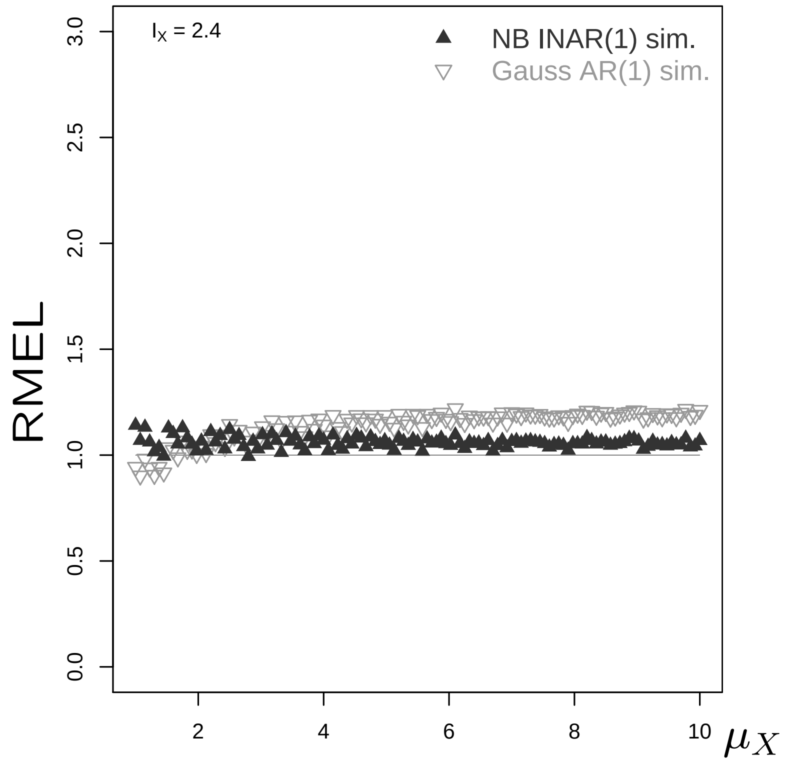


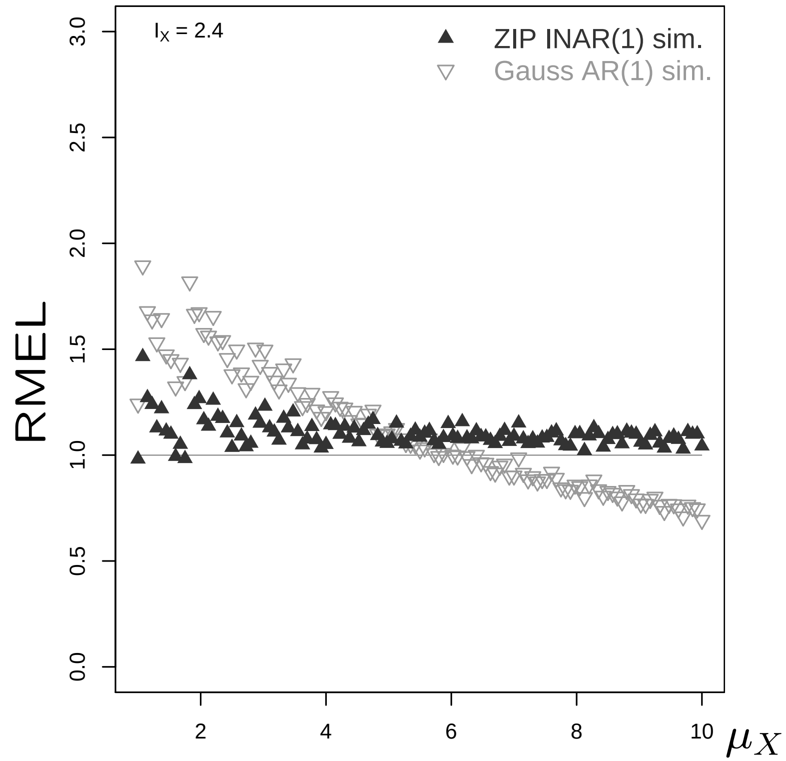



% of RMEL values $= 1$ (central area), $< 1$ (lower area) and $> 1$ (upper area) for different $T$. Grey area: Poi-INAR(1) forecasts, areas separated by black line: Gaussian AR(1) forecasts.



Mean of simulated RMEL values for NB-INAR(1) (left) and ZIP-INAR(1) (right), with $\alpha = 0.55$, $T = 250$, and $\sigma_X^2/\mu_X = 2.4$.



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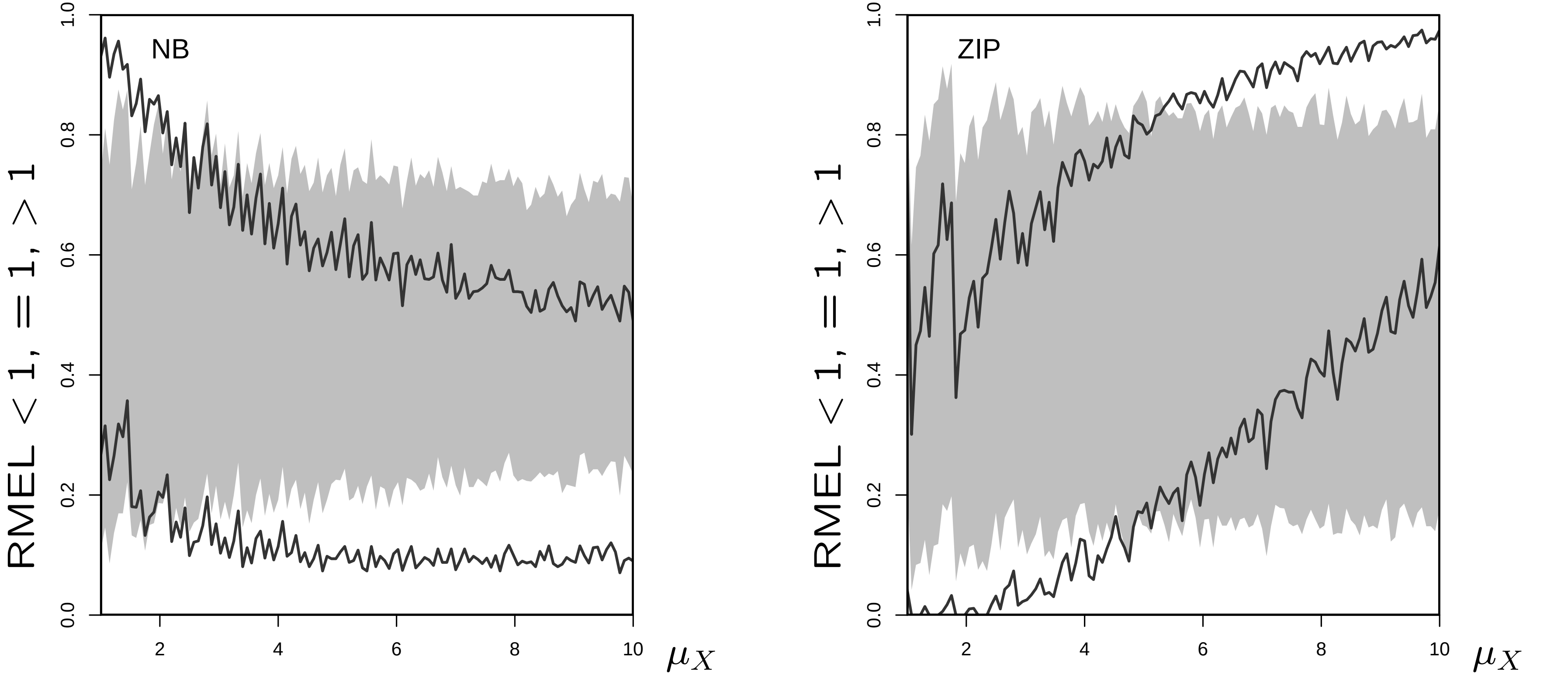
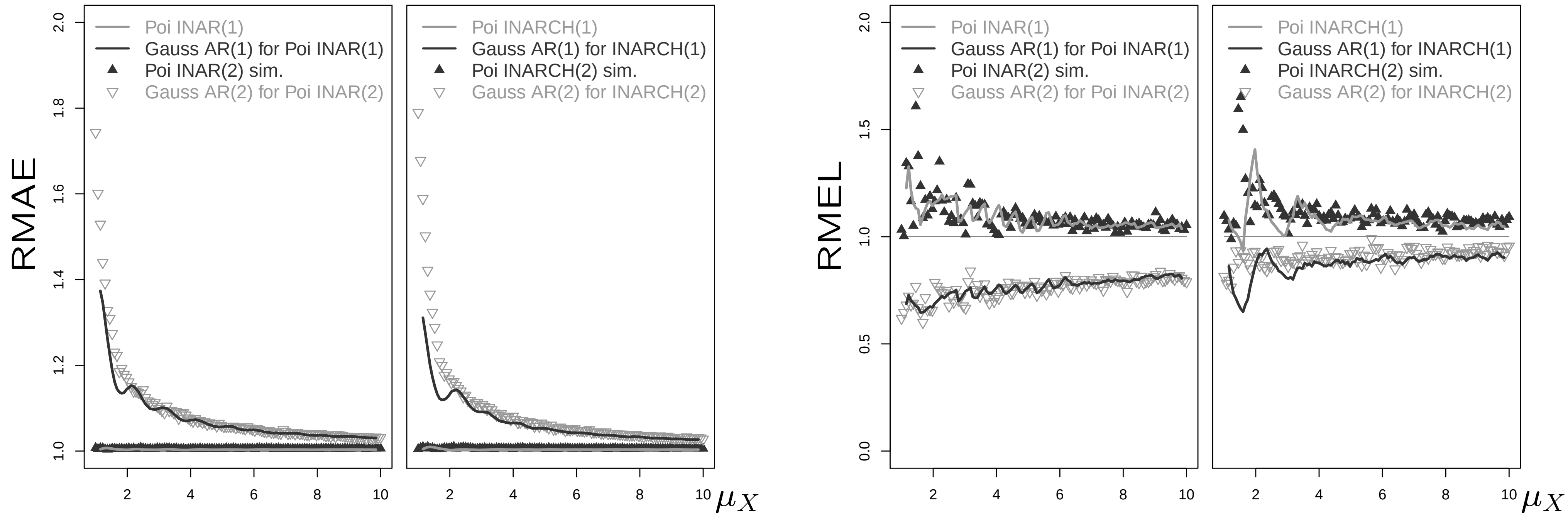
% of RMEL values $= 1$, $< 1$ and $> 1$ for NB-INAR(1) (left) and ZIP-INAR(1) (right). Grey area: INAR(1) forecasts, areas separated by black line: Gaussian AR(1) forecasts.



Mean RMAE (left) and RMEL (right) for DGP Poi-INAR(2) vs. Poi-INARCH(2), with $\alpha = 0.55$, $\alpha_2 = 0.45$ and $T = 250$; solid lines smoothed values of Poi-INAR(1) or Poi-INARCH(1) case.



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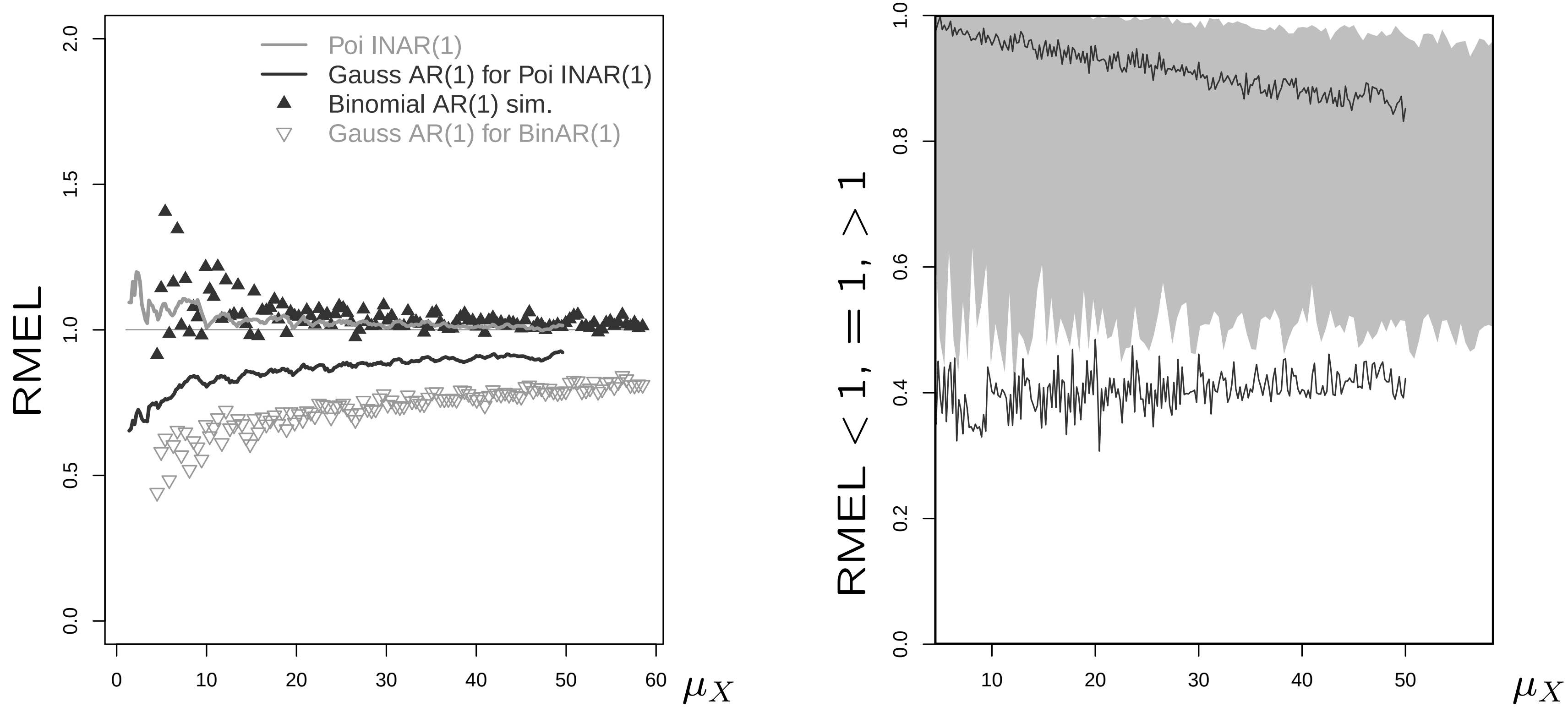
Mean RMEL (left) and % of RMEL values $= 1$, $< 1$ and $> 1$ (right) for BinAR(1) with $\pi = 0.45$, $\alpha = 0.55$, and $T = 250$.

Solid lines correspond to smoothed values of Poi-INAR(1) case.



## Conclusions



- Coherent point forecasting based on count model: estimation error nearly no effect on median forecasts; RMEL performance balanced and improves with increasing $T$.
- Point forecasting via Gaussian ARMA approximation: RMAE and RMEL performance considerably worse, strongly biased RMEL performance, severely affected by overdispersion or zero inflation, or by bounded counts.
- **Practice of discretizing Gaussian ARIMA forecasts for count time series strongly discouraged!**
- **Ongoing research:** construction and evaluation of (approximate) prediction intervals for count time series.

# Thank You

# for Your Interest!

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This research was funded by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) – Projektnummer 394832307.