EWMA Control Charts for Monitoring Binary Processes with Applications to Medical Diagnosis Data

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Some introductory words . . .

This talk is based on the paper

Weiß & Atzmüller (2009): EWMA Control Charts for Monitoring Binary Processes with Applications to Medical Diagnosis Data. Manuscript, 2009.
See conference CD-ROM.

All references mentioned in this talk correspond to the references in this article.

Medical Diagnosis Data

Motivation

Medical Diagnosis Data

Data sets from diagnostic expert system SonoConsult described by Huettig et al. (2004).

SonoConsult:

Medical documentation system for sonographic findings.

Used in DRK hospital in Berlin/Köpenick and in University hospital Würzburg.

Documents an average of about 300 cases per month in each clinic. (Atzmüller et al., 2005)

Cases consist of description of performed examinations, together with inferred diagnoses.

We extracted **time series of case records** belonging to certain diagnosis d and examiner e, with binary range {'0', '1'}, where:

'0' = 'considered diagnosis unclear or excluded',

'1' = 'considered diagnosis cannot be excluded'

$\Rightarrow$ '1' requires to take suitable countermeasures!

Basic assumption:

Binary time series of diagnosis d and examiner e, steming from underlying stochastic process $({X_t}^{(d,e)})_{\mathbb{N}}$ with possibly time-dependent marginal distribution: $P({X_t}^{(d,e)} = 1) =: {p_t}^{(d,e)}$.

From practical experience, even more restrictions expected:

. . .

If patients arrive independently of each other at examiner
$\Rightarrow ({X_t}^{(d,e)})_{\mathbb{N}}$ serially independent.

If, in addition, behavior of examiner does not change any more with time
$\Rightarrow ({X_t}^{(d,e)})_{\mathbb{N}}$ even i.i.d.
with time-independent marginal distribution $p^{(d,e)}$.

In-control model:
i.i.d. binary process with (unknown) probability $p_0^{(d,e)}$.

Aim: Provide semi-automatic component for online monitoring of behavior of different examiners.

Available time series data helps to investigate following **three major questions:**

- After evaluating norm profile of certain examiner and diagnosis (i. e., $p^{(d,e)}$): If same examiner decides again on specified diagnosis after longer break (e. g., job rotation), is marginal distribution still determined by $p^{(d,e)}$?
  $\rightarrow$ Typical phase I–phase II situation.

- Given norm profile of expert for diagnosis d (i. e. $p^{(d,0)}$): Does another examiner e deviate from norm profile, i. e., ${p_t}^{(d,e)} \neq p^{(d,0)}$?
  $\rightarrow$ Typical phase II situation.

- Without available knowledge (e. g., new examiner d): Is $({X_t}^{(d,e)})_{\mathbb{N}}$ stationary, or does distribution change with time (e. g., learning)?
  $\rightarrow$ Non-standard situation.

$\Rightarrow$ Require control chart concept for **continuous monitoring of i.i.d. binary processes**, which is **sufficiently flexible** to be adapted to three major problems outlined above!

p- and np-charts: data collected in samples.

Charts based on runs: well-suited for very small in-control probability p_0 (high quality processes).
But for diagnosis data, p_0 typically of medium size, such as 0.1 to 0.4.

EWMA Estimation and Monitoring of Probabilities

Background

Let $(X_t)_{\mathbb{N}}$ be i.i.d. binary process with $p := P(X_t = 1)$.

Sensitive EWMA Estimator:

Let $0 < \lambda < 1$ and $c \in [0;1]$, define

$$\hat{\pi}_t^{(\lambda)} \;=\; (1-\lambda)\cdot\hat{\pi}_{t-1}^{(\lambda)} \;+\; \lambda\cdot X_t \qquad \text{for } t \geq 1, \qquad \hat{\pi}_0^{(\lambda)} := c.$$

Properties:

$$E[\hat{\pi}_t^{(\lambda)}] \;=\; p \;+\; (1-\lambda)^t\cdot(c-p) \quad \underset{t\to\infty}{\longrightarrow} \quad p,$$

$$V[\hat{\pi}_t^{(\lambda)}] \quad \underset{t\to\infty}{\longrightarrow} \quad \tfrac{\lambda}{2-\lambda}\cdot p(1-p),$$

$$S[\hat{\pi}_t^{(\lambda)}] \quad \underset{t\to\infty}{\longrightarrow} \quad \tfrac{1-2p}{\sqrt{p(1-p)}}\cdot\tfrac{(2-\lambda)\cdot\sqrt{\lambda(2-\lambda)}}{3-3\lambda+\lambda^2}.$$

Basic EWMA Control of a Probability:

The sensitive statistics $\widehat{\pi}_t{}^{(\lambda)}$ with $\widehat{\pi}_0{}^{(\lambda)} := p_0$ are plotted on a chart with center line p_0 and control limits LCL_t and UCL_t.

Problem: How to choose control limits?

→ Require **simple design principle**, which is

easy to automatize, and

does not rely on trial-and-error (i. e., no simulations-based designs).

1st Idea: 3-σ limits

(Wheeler & Evandt, 2008, p. 207:

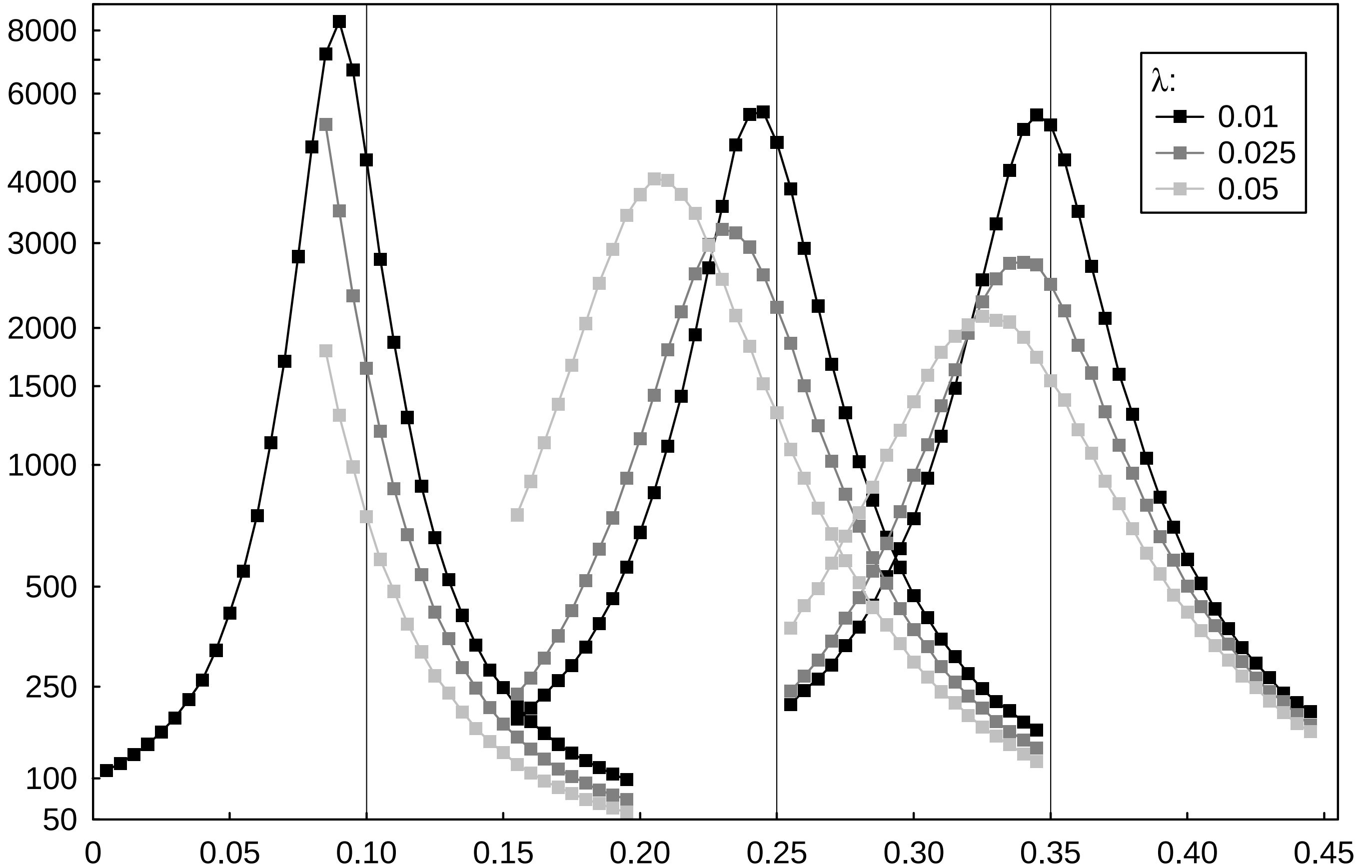
"a universal filter for separating the routine variation from any exceptional variation", which is *"not dependent on distributional assumptions in any critical way"*),

i. e.,

$$LCL_t/UCL_t \;:=\; p_0 \mp 3 \cdot \sqrt{V_0[\widehat{\pi}_t^{(\lambda)}]}.$$

But statistics $\widehat{\pi}_t^{(\lambda)}$ skewed!

$ARL(p)$ performance of EWMA charts with 3-σ limits and in-control probabilities $p_0 = 0.1, 0.25, 0.35$:



p

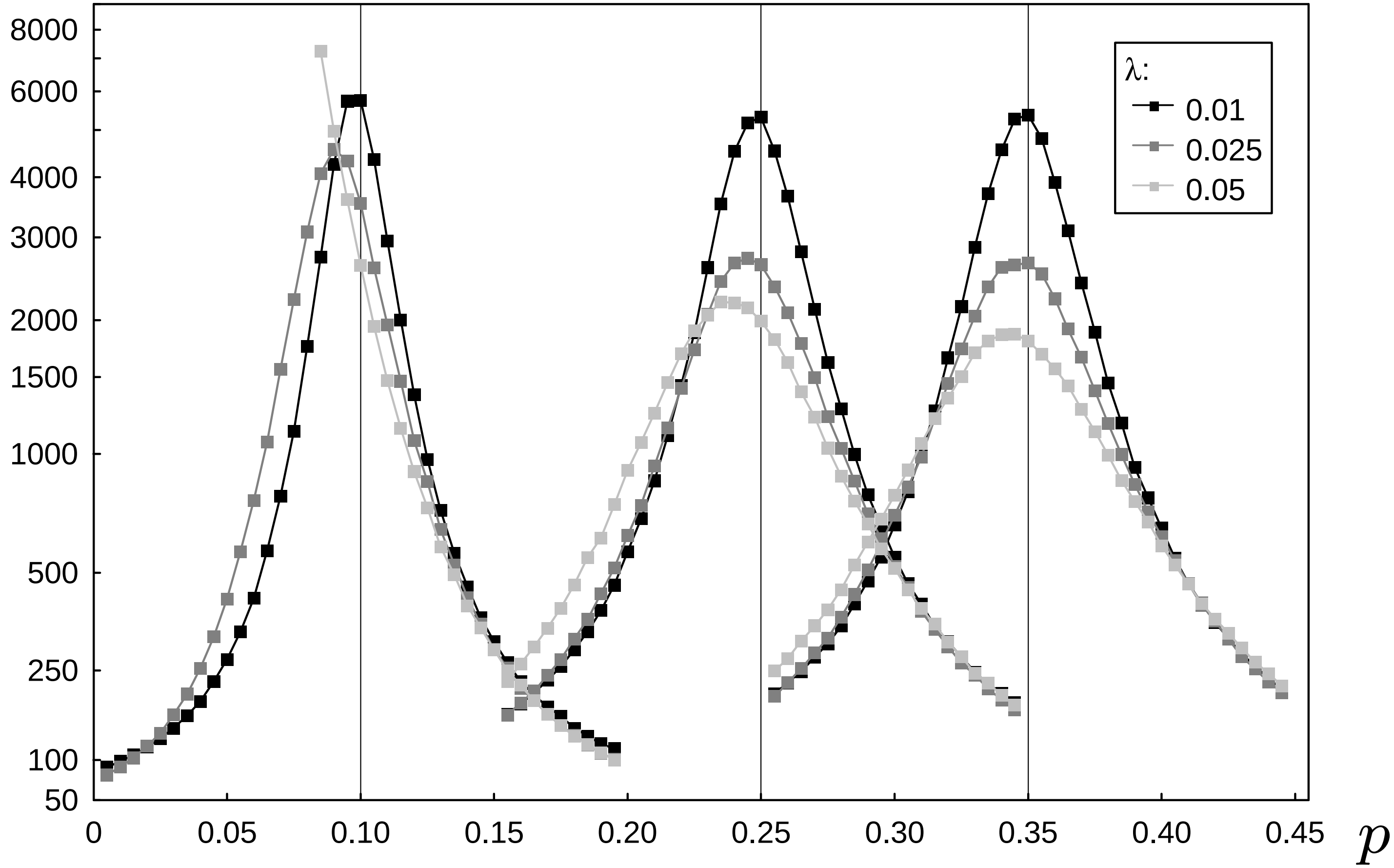
2$^{\text{nd}}$ Idea: Skewness-adjusted 3-σ control limits as proposed by Chan & Cui (2003):

$$LCL_t/UCL_t \;:=\; p_0 + (\mp 3 + c_4^*)\cdot\sqrt{V_0[\widehat{\pi}_t{}^{(\lambda)}]},$$

$$\text{where } c_4^* \;:=\; \tfrac{4}{3}\cdot S_0[\widehat{\pi}_t{}^{(\lambda)}]\cdot\Big(1+0.2\cdot S_0^2[\widehat{\pi}_t{}^{(\lambda)}]\Big)^{-1}.$$

Only additional correction term, easily implemented in practice!

$ARL(p)$ performance of EWMA charts with skewness-adjusted 3-σ limits and in-control probabilities $p_0 = 0.1, 0.25, 0.35$:



In the following, we adapt basic EWMA Scheme to situations with different degree of knowledge about true in-control probability p_0.

All control charts are designed with skewness-adjusted 3-σ limits based on asymptotic variance and skewness provided above.

EWMA Chart with Estimated p_0

1^{st} Situation

EWMA Chart with Estimated p_0

Standard phase I situation, relevant if
examiner has worked on diagnosis for longer time.

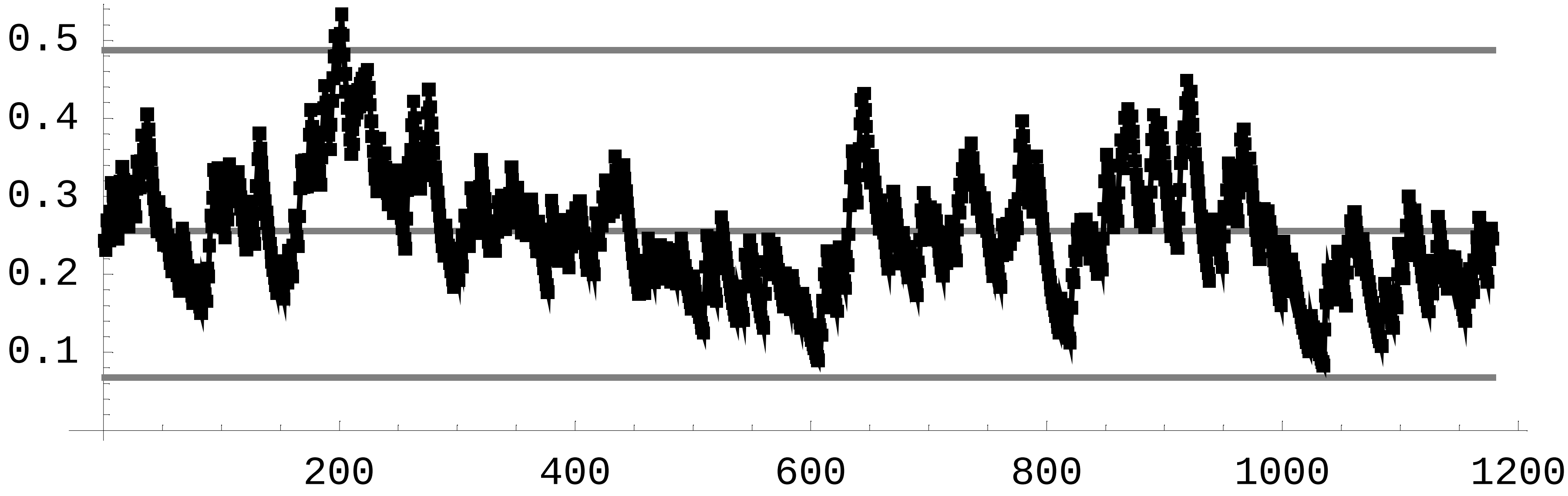
Example:

Diagnosis $d =$'M165' (fatty liver), examiner $e =$'Mf542a5'.

Cases x_1 to x_{1178}:
connected period of time (Feb. 2002 to Sep. 2003).
Then break of about 8 months.
Cases x_{1179} to x_{1627}:
many interruptions (May 2004 to June 2007).

EWMA Chart with Estimated p_0

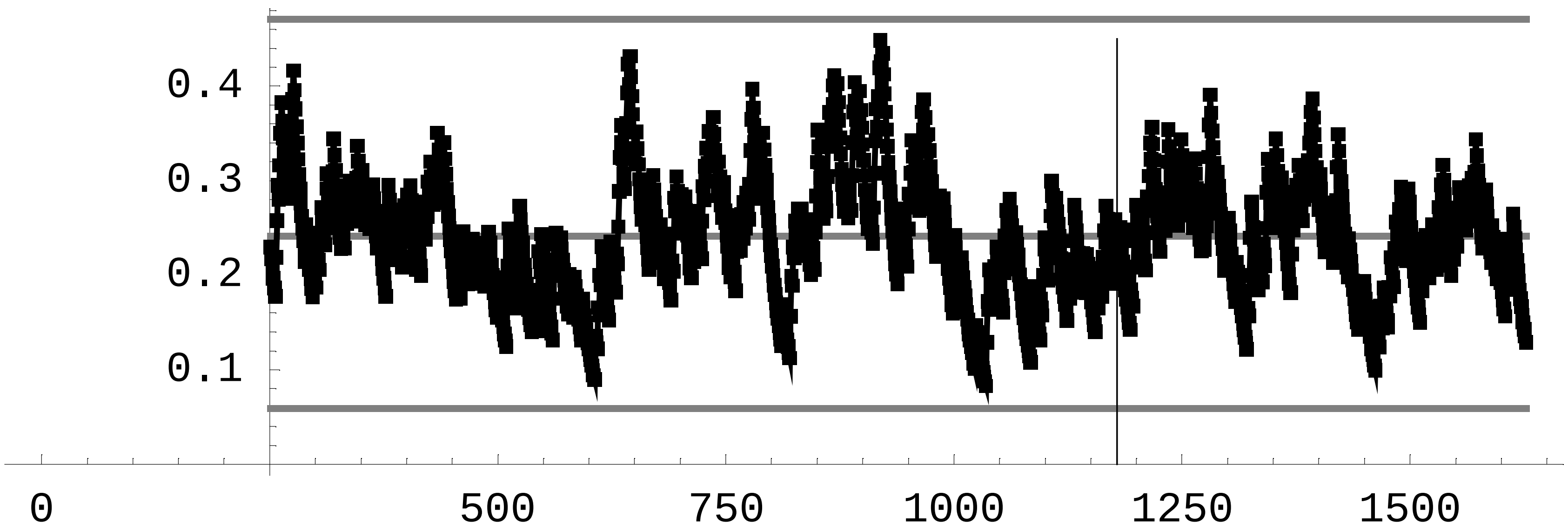
Data of period $t = 1$ to $t = 1178$, estimate $\widehat{p}_0 = 0.255518$.



Out-of-control situation around $t = 200$,

but afterwards process varies at constant level.

Data between $t = 251$ and $t = 1178$, estimate $\widehat{p}_0 = 0.241379$.



No points plotted beyond limits,

examiner seems in-control also for $t \geq 1179$.

EWMA Chart with Given p_0

2nd Situation

EWMA Chart with Given p_0

Standard phase II situation, relevant if reference value p_0 known from expert or literature.

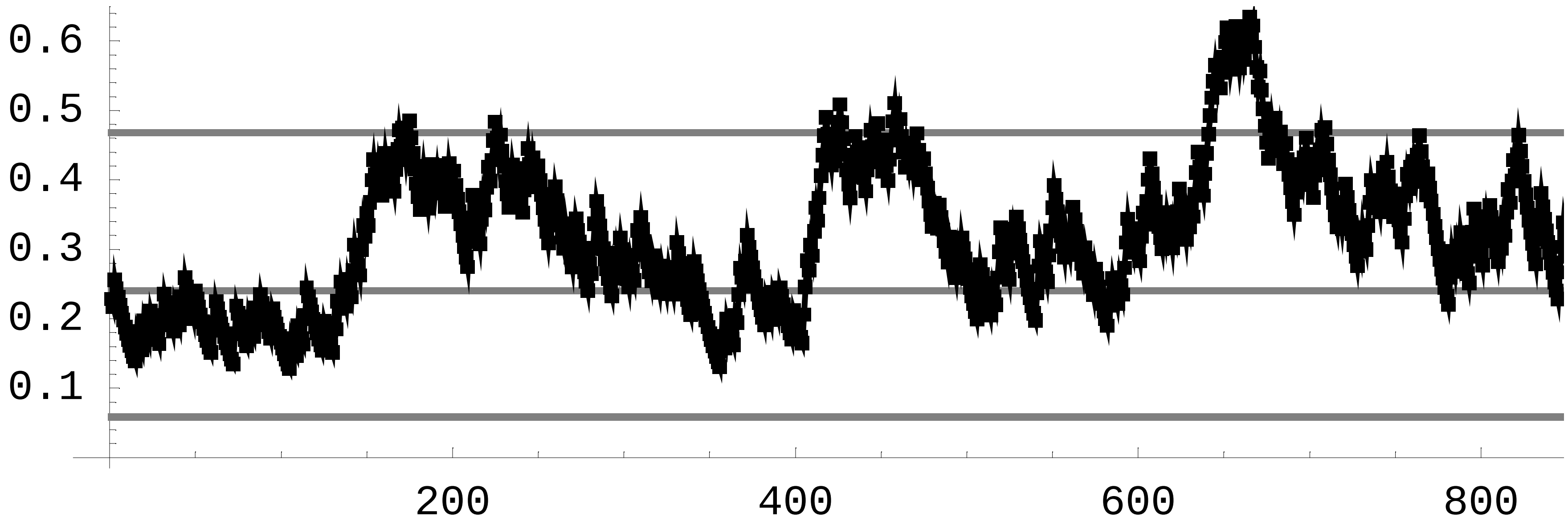
Example:

Expert $e =$'Mf542a5' above for diagnosis $d =$'M165' (fatty liver), providing reference distribution $p_0 = 0.24$.

Consider examiner $e =$'Mf542a9' with cases x_1 to x_{856} steming from connected period of time (Apr. 2004 to Dec. 2004).

EWMA Chart with Given p_0

Phase II EWMA control chart ($\lambda = 0.05$) for data Mf542a9 (M165) with $p_0 = 0.24$:



Starts with constant probability around p_0, but then shifts to higher level after time $t = 150$.

EWMA Chart without Knowledge about p_0

3rd Situation

EWMA Chart without Knowledge about p_0

Task: Monitor $(X_t)_{\mathbb{N}}$ *without* having *any prior information* on distribution.

Past experience:

$(X_t)_{\mathbb{N}}$ usually roughly stationary in beginning (with unknown p_0) and changes almost slowly with time (learning).

Idea:

Use first observations **also** to continuously estimate p_0 with estimator being more robust than plotted $\widehat{\pi}_t^{(\lambda)}$.

EWMA estimator $\widehat{p}_t{}^{(\lambda)} := \frac{1}{t} \cdot \sum_{i=1}^{t} \widehat{\pi}_i{}^{(\lambda)}$ for $t \geq 1$ is much more robust than $\widehat{\pi}_t^{(\lambda)}$: $\quad V[\widehat{p}_t^{(\lambda)}] \quad \underset{t\to\infty}{\longrightarrow} \quad 0.$

EWMA Chart without Knowledge about p_0

But $\widehat{p}_t^{(\lambda)}$ very robust already in beginning:

If initial value c far away from true p_0, then many alarms.

$\Rightarrow$ Use variable smoothing parameter λ_t,

such that

resulting estimator moderately robust in beginning,

and more and more robust with time t.

EWMA Chart without Knowledge about p_0

Let $\tilde{\lambda}$ and $\lambda_0 > \lambda_\infty$ be from $(0;1)$, define

$$\lambda_t \;=\; (1-\tilde{\lambda})\cdot\lambda_{t-1} \;+\; \tilde{\lambda}\cdot\lambda_\infty.$$

So λ_t converges to λ_∞ (chosen sufficiently small), but for small t, λ_t is larger than λ_∞
$\Rightarrow$ $\widehat{p}_t{}^{(\lambda_t)}$ quickly tends to value near p_0 in beginning.

Estimator $\widehat{p}_t{}^{(\lambda_t)}$ computed recursively according to

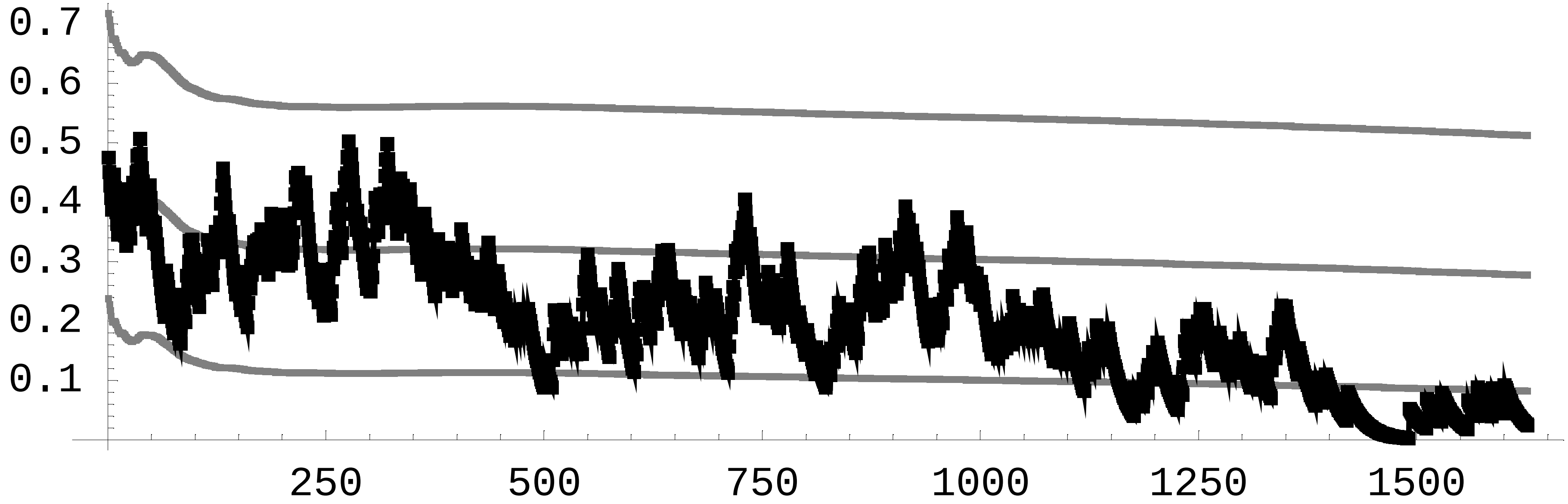
$$Z_t^{(\lambda_t)} := (1-\lambda_t)\cdot Z_{t-1}^{(\lambda_{t-1})} + \lambda_t\cdot X_t, \qquad p_t^{(\lambda_t)} := \tfrac{1}{t}\cdot\sum_{i=1}^{t} Z_i^{(\lambda_i)},$$

With $\lambda \in (0;1)$, the sensitive statistics $\widehat{\pi}_t{}^{(\lambda)}$ are plotted on chart with time-varying control limits LCL_t and UCL_t based on $\widehat{p}_t{}^{(\lambda_t)}$. $\quad$ ($\rightarrow$ **self-learning EWMA chart**)

Example:

Examiner $e =$'Mf542a5' for diagnosis $d =$'M162' (cirrhosis of the liver). Design parameters $c = 0.5$, $\lambda = \lambda_0 = 0.05$, $\lambda_\infty = 0.001$, $\tilde{\lambda} = 0.01$.

EWMA Chart without Knowledge about p_0



Initial level ≈ 0.35, quickly learned by EWMA chart.

$\lambda_\infty = 0.001$ very small, so limits very robust after $t = 200$.

Between $t = 400$ and $t = 500$, process level shifts down to about 0.25, and decreases again after $t = 1000$.

All these changes signaled by **self-learning EWMA chart**!

EWMA Charts for Monitoring Binary Processes

Conclusions

Conclusions

- EWMA control charts for monitoring binary processes for situations with different degree of prior knowledge about true in-control probability p_0;
- skewness-corrected 3-σ limits lead to better ARL performance than standard 3-σ limits;
- case study utilizing medical diagnosis data from the real-world system SonoConsult;
- EWMA charts easily designed and automatized, successfully applied in any situation.

Thank You

for Your Interest!

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