EWMA Control Charts for Monitoring Binary Processes with Applications to Medical Diagnosis Data

Christian H. Weiβ

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Department of Mathematics,
Darmstadt University of Technology

Martin Atzmüller
Institute of Computer Science,
University of Würzburg
Some introductory words . . .

This talk is based on the paper


*See conference CD-ROM.*

All references mentioned in this talk correspond to the references in this article.

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Medical Diagnosis Data

Motivation
Medical Diagnosis Data

Data sets from diagnostic expert system SonoConsult described by Huettig et al. (2004).

SonoConsult:
Medical documentation system for sonographic findings. Used in DRK hospital in Berlin/Köpenick and in University hospital Würzburg. Documents an average of about 300 cases per month in each clinic. (Atzmüller et al., 2005)
Medical Diagnosis Data

Cases consist of description of performed examinations, together with inferred diagnoses.

We extracted **time series of case records** belonging to certain diagnosis $d$ and examiner $e$, with binary range $\{\text{‘0’, ‘1’}\}$, where:

‘0’ = ‘considered diagnosis unclear or excluded’,
‘1’ = ‘considered diagnosis cannot be excluded’

$\Rightarrow$ ‘1’ requires to take suitable countermeasures!
Medical Diagnosis Data

**Basic assumption:**

Binary time series of diagnosis $d$ and examiner $e$, stemming from underlying stochastic process $(X_t^{(d,e)})_N$ with possibly time-dependent marginal distribution:

$$P(X_t^{(d,e)} = 1) =: p_t^{(d,e)}.$$  

From practical experience, even more restrictions expected:

\[ \ldots \]
If patients arrive independently of each other at examiner
⇒ \((X_t^{(d,e)})_N\) serially independent.

If, in addition, behavior of examiner does not change any more with time
⇒ \((X_t^{(d,e)})_N\) even i.i.d.
with time-independent marginal distribution \(p^{(d,e)}\).

**In-control model:**
i.i.d. binary process with (unknown) probability \(p_0^{(d,e)}\).
Medical Diagnosis Data

**Aim:** Provide semi-automatic component for online monitoring of behavior of different examiners.

Available time series data helps to investigate following **three major questions:**

- After evaluating norm profile of certain examiner and diagnosis (i.e., $p^{(d,e)}$): If same examiner decides again on specified diagnosis after longer break (e.g., job rotation), is marginal distribution still determined by $p^{(d,e)}$? → Typical phase I–phase II situation.

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• Given norm profile of expert for diagnosis $d$ (i.e. $p^{(d,0)}$): Does another examiner $e$ deviate from norm profile, i.e.,
$p_{t}^{(d,e)} \neq p^{(d,0)}$?
→ Typical phase II situation.

• Without available knowledge (e.g., new examiner $d$):
Is $(X_{t}^{(d,e)})_{N}$ stationary, or does distribution change with time (e.g., learning)?
→ Non-standard situation.
⇒ Require control chart concept for **continuous monitoring of i.i.d. binary processes**, which is **sufficiently flexible** to be adapted to three major problems outlined above!

\( p \)- and \( np \)-charts: data collected in samples.

Charts based on runs: well-suited for very small in-control probability \( p_0 \) (high quality processes).

But for diagnosis data, \( p_0 \) typically of medium size, such as 0.1 to 0.4.

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EWMA Estimation and Monitoring of Probabilities

Background
Let \((X_t)_{N}\) be i.i.d. binary process with \(p := P(X_t = 1)\).

**Sensitive EWMA Estimator:**

Let \(0 < \lambda < 1\) and \(c \in [0; 1]\), define

\[
\hat{\pi}_t^{(\lambda)} = (1 - \lambda) \cdot \hat{\pi}_{t-1}^{(\lambda)} + \lambda \cdot X_t \quad \text{for } t \geq 1, \quad \hat{\pi}_0^{(\lambda)} := c.
\]

**Properties:**

\[
E[\hat{\pi}_t^{(\lambda)}] = p + (1 - \lambda)^t \cdot (c - p) \xrightarrow{t \to \infty} p,
\]

\[
V[\hat{\pi}_t^{(\lambda)}] \xrightarrow{t \to \infty} \frac{\lambda}{2 - \lambda} \cdot p(1 - p),
\]

\[
S[\hat{\pi}_t^{(\lambda)}] \xrightarrow{t \to \infty} \frac{1 - 2p}{\sqrt{p(1 - p)}} \cdot \frac{(2 - \lambda) \cdot \sqrt{\lambda(2 - \lambda)}}{3 - 3\lambda + \lambda^2}.
\]
Basic EWMA Control of a Probability:
The sensitive statistics $\hat{\pi}_t(\lambda)$ with $\hat{\pi}_0(\lambda) := p_0$ are plotted on a chart with center line $p_0$ and control limits $LCL_t$ and $UCL_t$.

**Problem:** How to choose control limits?
→ Require **simple design principle**, which is easy to automatize, and does not rely on trial-and-error (i.e., no simulations-based designs).
1st Idea: 3-σ limits

(Wheeler & Evandt, 2008, p. 207: “a universal filter for separating the routine variation from any exceptional variation”, which is “not dependent on distributional assumptions in any critical way”), i. e.,

\[ LCL_t/UCL_t := p_0 \mp 3 \cdot \sqrt{V_0[\hat{\pi}_t^{(\lambda)}]} \].

But statistics \( \hat{\pi}_t^{(\lambda)} \) skewed!

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$ARL(p)$ performance of EWMA charts with 3-$\sigma$ limits and in-control probabilities $p_0 = 0.1, 0.25, 0.35$: 

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{chart.png}
\end{figure}
2\textsuperscript{nd} Idea: Skewness-adjusted 3-\(\sigma\) control limits as proposed by Chan & Cui (2003):

\[
LCL_t/UCL_t := p_0 + (\mp 3 + c_4^*) \cdot \sqrt{V_0[\hat{\pi}_t(\lambda)]},
\]

where \(c_4^* := \frac{4}{3} \cdot S_0[\hat{\pi}_t(\lambda)] \cdot (1 + 0.2 \cdot S_0^2[\hat{\pi}_t(\lambda)])^{-1}.

Only additional correction term, easily implemented in practice!
ARL($p$) performance of EWMA charts with skewness-adjusted 3-$\sigma$ limits and in-control probabilities $p_0 = 0.1, 0.25, 0.35$.
In the following, we adapt basic EWMA Scheme to situations with different degree of knowledge about true in-control probability $p_0$.

All control charts are designed with skewness-adjusted $3-\sigma$ limits based on asymptotic variance and skewness provided above.
EWMA Chart with Estimated $p_0$
Standard phase I situation, relevant if examiner has worked on diagnosis for longer time.

Example:
Diagnosis $d = \text{‘M165’ (fatty liver)}$, examiner $e = \text{‘Mf542a5’}$.

Cases $x_1$ to $x_{1178}$: connected period of time (Feb. 2002 to Sep. 2003).
Then break of about 8 months.
Cases $x_{1179}$ to $x_{1627}$: many interruptions (May 2004 to June 2007).
Data of period $t = 1$ to $t = 1178$, estimate $\hat{p}_0 = 0.255518$.

Out-of-control situation around $t = 200$, but afterwards process varies at constant level.
EWMA Chart with Estimated $p_0$

Data between $t = 251$ and $t = 1178$, estimate $\hat{p}_0 = 0.241379$.

No points plotted beyond limits, examiner seems in-control also for $t \geq 1179$.  

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EWMA Chart with Given $p_0$

2\textsuperscript{nd} Situation
Standard phase II situation, relevant if reference value \( p_0 \) known from expert or literature.

**Example:**

Expert \( e = 'Mf542a5' \) above for diagnosis \( d = 'M165' \) (fatty liver), providing reference distribution \( p_0 = 0.24 \).

Consider examiner \( e = 'Mf542a9' \) with cases \( x_1 \) to \( x_{856} \) stemming from connected period of time (Apr. 2004 to Dec. 2004).
Phase II EWMA control chart ($\lambda = 0.05$) for data Mf542a9 (M165) with $p_0 = 0.24$:

Starts with constant probability around $p_0$, but then shifts to higher level after time $t = 150$. 

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EWMA Chart without Knowledge about $p_0$

3rd Situation
Task: Monitor \((X_t)_N\) without having any prior information on distribution.

Past experience:
\((X_t)_N\) usually roughly stationary in beginning (with unknown \(p_0\)) and changes almost slowly with time (learning).

Idea:
Use first observations also to continuously estimate \(p_0\) with estimator being more robust than plotted \(\hat{\pi}_t(\lambda)\).

EWMA estimator \(\hat{p}_t(\lambda) := \frac{1}{t} \cdot \sum_{i=1}^{t} \hat{\pi}_i(\lambda)\) for \(t \geq 1\) is much more robust than \(\hat{\pi}_t(\lambda)\):
\[
V[\hat{p}_t(\lambda)] \xrightarrow{t \to \infty} 0.
\]
But $\hat{p}_t(\lambda)$ very robust already in beginning:

If initial value $c$ far away from true $p_0$, then many alarms.

⇒ Use variable smoothing parameter $\lambda_t$,
such that
resulting estimator moderately robust in beginning,
and more and more robust with time $t$. 
Let $\tilde{\lambda}$ and $\lambda_0 > \lambda_\infty$ be from $(0; 1)$, define

$$\lambda_t = (1 - \tilde{\lambda}) \cdot \lambda_{t-1} + \tilde{\lambda} \cdot \lambda_\infty.$$ 

So $\lambda_t$ converges to $\lambda_\infty$ (chosen sufficiently small), but for small $t$, $\lambda_t$ is larger than $\lambda_\infty$

$\Rightarrow \hat{p}_t(\lambda_t)$ quickly tends to value near $p_0$ in beginning.
EWMA Chart without Knowledge about $p_0$

Estimator $\hat{p}_t(\lambda_t)$ computed recursively according to

$$Z_t(\lambda_t) := (1-\lambda_t)Z_{t-1}(\lambda_{t-1}) + \lambda_t \cdot X_t,$$

$$p_t(\lambda_t) := \frac{1}{t} \sum_{i=1}^{t} Z_i(\lambda_i),$$

With $\lambda \in (0; 1)$, the sensitive statistics $\hat{\pi}_t(\lambda)$ are plotted on chart with time-varying control limits $LCL_t$ and $UCL_t$ based on $\hat{p}_t(\lambda_t)$. (→ self-learning EWMA chart)

**Example:**

Examiner $e = 'Mf542a5'$ for diagnosis $d = 'M162'$ (cirrhosis of the liver). Design parameters $c = 0.5$, $\lambda = \lambda_0 = 0.05$, $\lambda_\infty = 0.001$, $\bar{\lambda} = 0.01$. 

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Initial level $\approx 0.35$, quickly learned by EWMA chart.

$\lambda_\infty = 0.001$ very small, so limits very robust after $t = 200$.

Between $t = 400$ and $t = 500$, process level shifts down to about 0.25, and decreases again after $t = 1000$.

All these changes signaled by **self-learning EWMA chart**!
EWMA Charts for Monitoring Binary Processes

Conclusions
Conclusions

• EWMA control charts for monitoring binary processes for situations with different degree of prior knowledge about true in-control probability $p_0$;

• skewness-corrected 3-$\sigma$ limits lead to better $ARL$ performance than standard 3-$\sigma$ limits;

• case study utilizing medical diagnosis data from the real-world system SonoConsult;

• EWMA charts easily designed and automatized, successfully applied in any situation.

Christian H. Weiβ — Darmstadt University of Technology
Thank You
for Your Interest!

Christian H. Weiß
Department of Mathematics
Darmstadt University of Technology
weiss@mathematik.tu-darmstadt.de