Group Inspection of Dependent Binary Processes.

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See conference CD-ROM.

All references mentioned in this talk correspond to the references in this article.

Dependent Binary Processes

Introduction

Statistical control of binary attribute processes $(X_t)_{\mathbb{N}}$ of particular relevance both in research and professional practice.

Typical example: Result of an inspection of an item.
$X_t = 1$ iff the item was *defective* or *nonconforming*.

Many control charts if $(X_t)_{\mathbb{N}}$ is monitored continuously, e. g.:

- run length (RL) charts of Bourke (1991), $(X_t)_{\mathbb{N}}$ serially independent.

- geometric CUSUM procedure of Bourke (1991), $(X_t)_{\mathbb{N}}$ serially independent.

- modified RL charts if $(X_t)_{\mathbb{N}}$ exhibits serial dependence of Markov type, see Blatterman & Champ (1992), Lai et al. (2000), Shepherd et al. (2007).

In practice, often impossible to monitor process continuously:

- Evaluation of items may be too expensive,
- evaluation of items may even destroy items,
- evaluation takes too much time (relative to process speed),
- evaluation complex (e. g., endurance or tolerance tests).

Examples of high-speed processes also outside manufacturing industry:

Lambert & Liu (2006) monitored and controlled a communication network by considering packets handled or dropped, processing errors of various types, and others.

Intrusion detection system described by Ye et al. (2001): controls sequences of counts (per unit time) of method requests or particular outcomes on the system.

In the following, assume that only segments of process $(X_t)_{\mathbb{N}}$ are analyzed, taken at times $t_1, t_2, \ldots$ with $t_k - t_{k-1}$ sufficiently large. Statistic: **segment sum** ${C_k}^{(n)} = X_{t_k} + \ldots + X_{t_k+n-1}$ (count of nonconforming items).

Many control charts if $(X_t)_{\mathbb{N}}$ serially independent, e. g.: p and np chart (Montgomery, 2005), Q chart (Quesenberry, 1991), EWMA chart (Gan, 1990), MA chart (Khoo, 2004), CUSUM chart (Gan, 1993).

Assumption of serial independence often violated in practice!

Assume that time distances $t_k - t_{k-1}$ sufficiently large such that segment sums ${C_k}^{(n)}$ are approximately i.i.d. Within samples, dependence structure of process has to be considered.

$\Rightarrow$ Marginal distribution of $({C_k}^{(n)})_{\mathbb{N}}$ in general not binomial.

Idea:

Use **Markov binomial distribution** $MB(n, p, \rho)$ to approximate true distribution of counts $C_k^{(n)}$.

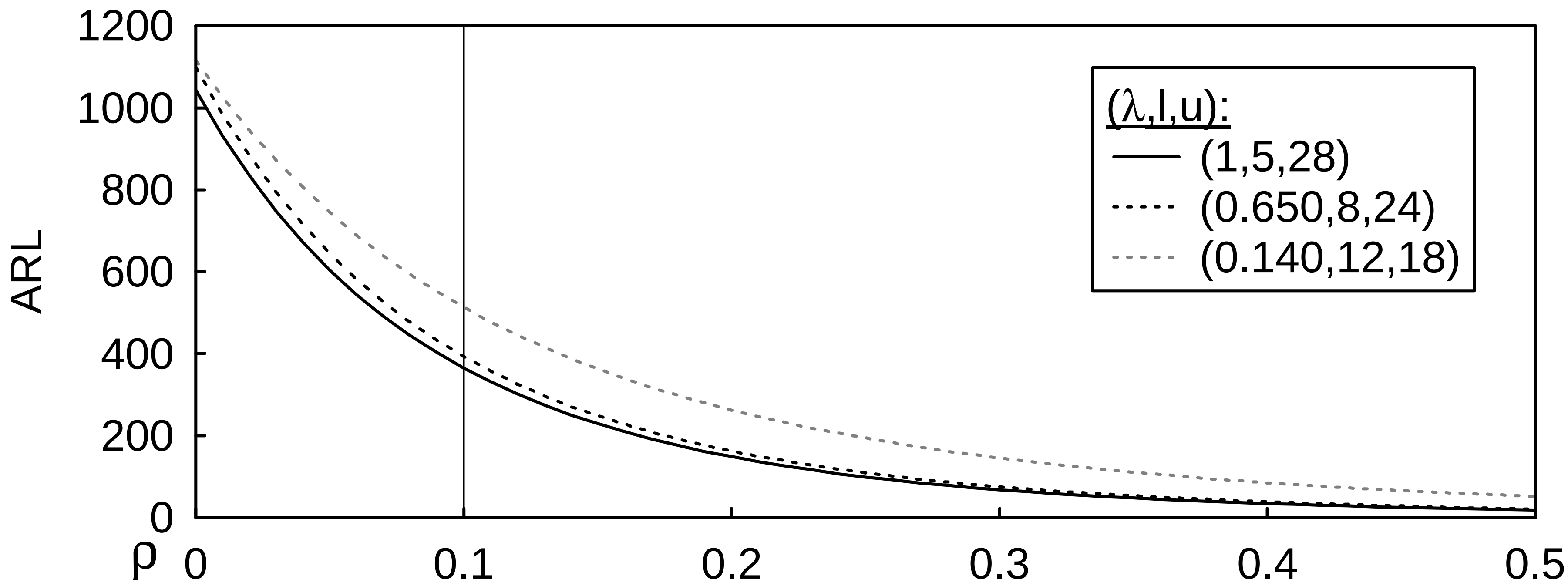
($MB(n, p, 0)$ distribution = binomial distribution $B(n, p)$.)

We consider np **chart** and **EWMA chart** of Gan (1990):

Setting $Z_0 = z_0$, it controls the statistics $(Z_k)_{\mathbb{N}}$ defined by

$$Z_k \;=\; \text{round}\big(\lambda \cdot C_k^{(n)} \;+\; (1-\lambda) \cdot Z_{k-1}\big), \qquad \lambda \in (0; 1].$$

np and EWMA chart very sensitive towards serial dependence, $ARL(\rho)$:
(example of Gan (1990), where $n = 150$, $p = p_0 = 0.1$)



Therefore:

Consider degree of autocorrelation to design chart appropriately:

$\Rightarrow$ **Markov np chart**

$\Rightarrow$ **Markov binomial EWMA chart**

The
Markov np Chart

Design & Performance

Markov np chart with 3-σ limits:

Counts $C_k^{(n)}$ plotted on chart with center line np_0 and control limits

$$\begin{aligned} LCL &= \max\left(0,\ np_0 - 3\sqrt{np_0(1-p_0)\cdot\frac{1+\rho_0}{1-\rho_0}\cdot\left(1-\frac{2\rho_0(1-\rho_0^n)}{n(1-\rho_0^2)}\right)}\right), \\ UCL &= \min\left(n,\ np_0 + 3\sqrt{np_0(1-p_0)\cdot\frac{1+\rho_0}{1-\rho_0}\cdot\left(1-\frac{2\rho_0(1-\rho_0^n)}{n(1-\rho_0^2)}\right)}\right). \end{aligned}$$

ARL **computation:**

$$ARL(n,p,\rho) \;=\; \left(1 \;-\; \textstyle\sum_{j=LCL}^{UCL}\; P(C^{(n)}=j)\right)^{-1},$$

using the $MB(n,p,\rho)$ distribution

In-control performance

Choice of 3-σ limits: ARL_0s vary heavily for varying p_0, ρ_0, often control even impossible ($LCL = 0$ and $UCL = n$).

Recommendations for design (LCL, UCL) such that $ARL \approx 300$ $\rightarrow$ see Weiß (2008).

Out-of-control performance

See below, since the Markov np chart can be considered as a special EWMA chart with $\lambda = 1$.

The
Markov Binomial
EWMA Chart

Design & Performance

The process $(Z_k)_{\mathbb{N}_0}$, defined by

$$Z_k \;=\; \text{round}\big(\lambda \cdot C_k^{(n)} \;+\; (1-\lambda)\cdot Z_{k-1}\big), \qquad \lambda \in (0;1],$$

is a homogeneous Markov chain with range $\{0,\ldots,n\}$ and transition probabilities

$$p(x|y) \;:=\; P(Z_k = x \mid Z_{k-1} = y)$$
$$=\; P\big(\tfrac{1}{\lambda}(x-0.5-(1-\lambda)y) \leq C_k^{(n)} < \tfrac{1}{\lambda}(x+0.5-(1-\lambda)y)\big).$$

$\Rightarrow$ ARLs can be computed exactly by adapting the Markov chain approach of Brook & Evans (1972).

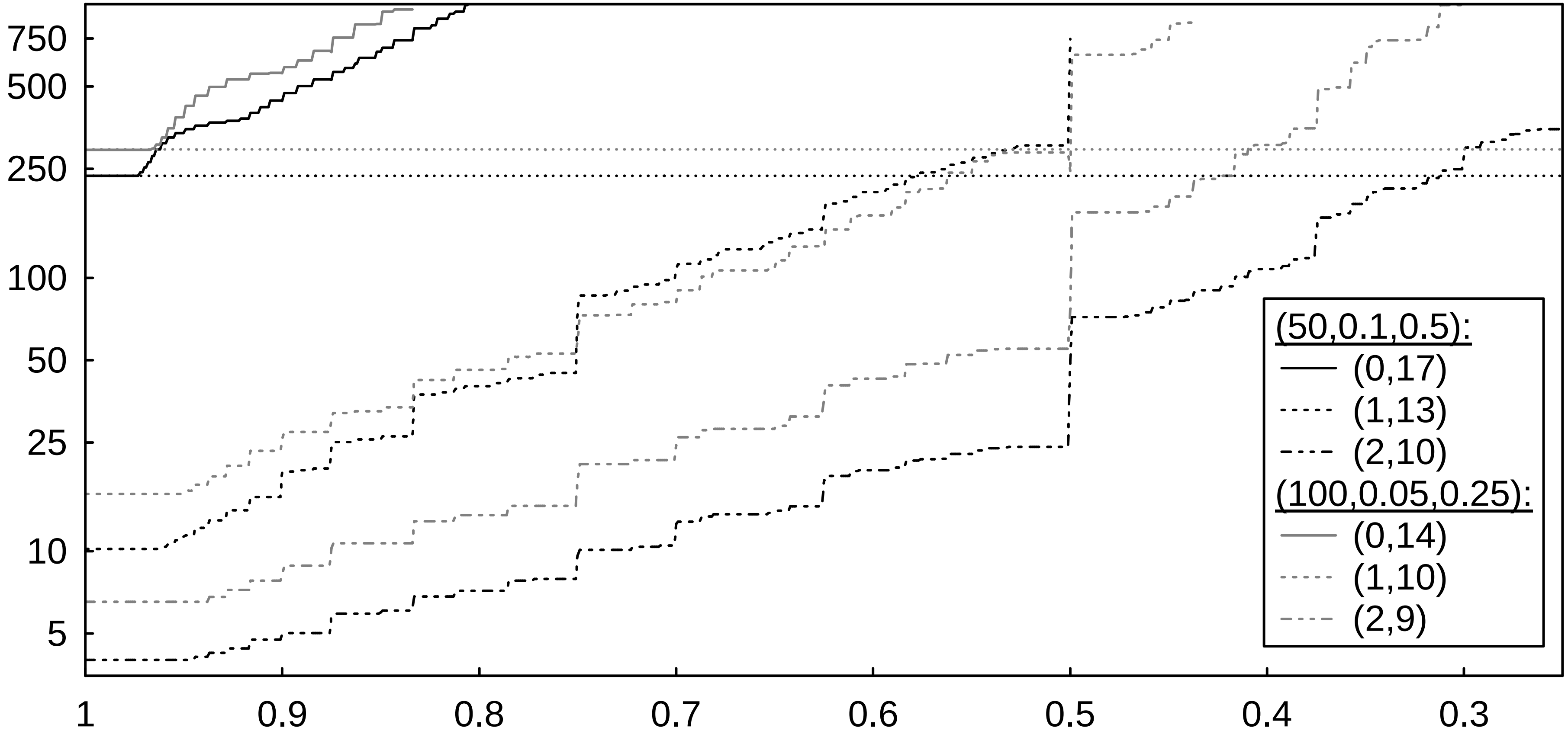
Using this approach, we recommend the following

Design strategy:

1. Start with Markov np chart (i. e.: $\lambda = 1$) and with sufficiently small ARL_0.

2. Once limits (l, u) have been fixed, then decreasing λ increases ARL_0.
   If desired ARL_0 reached, the design (λ, l, u) is complete.

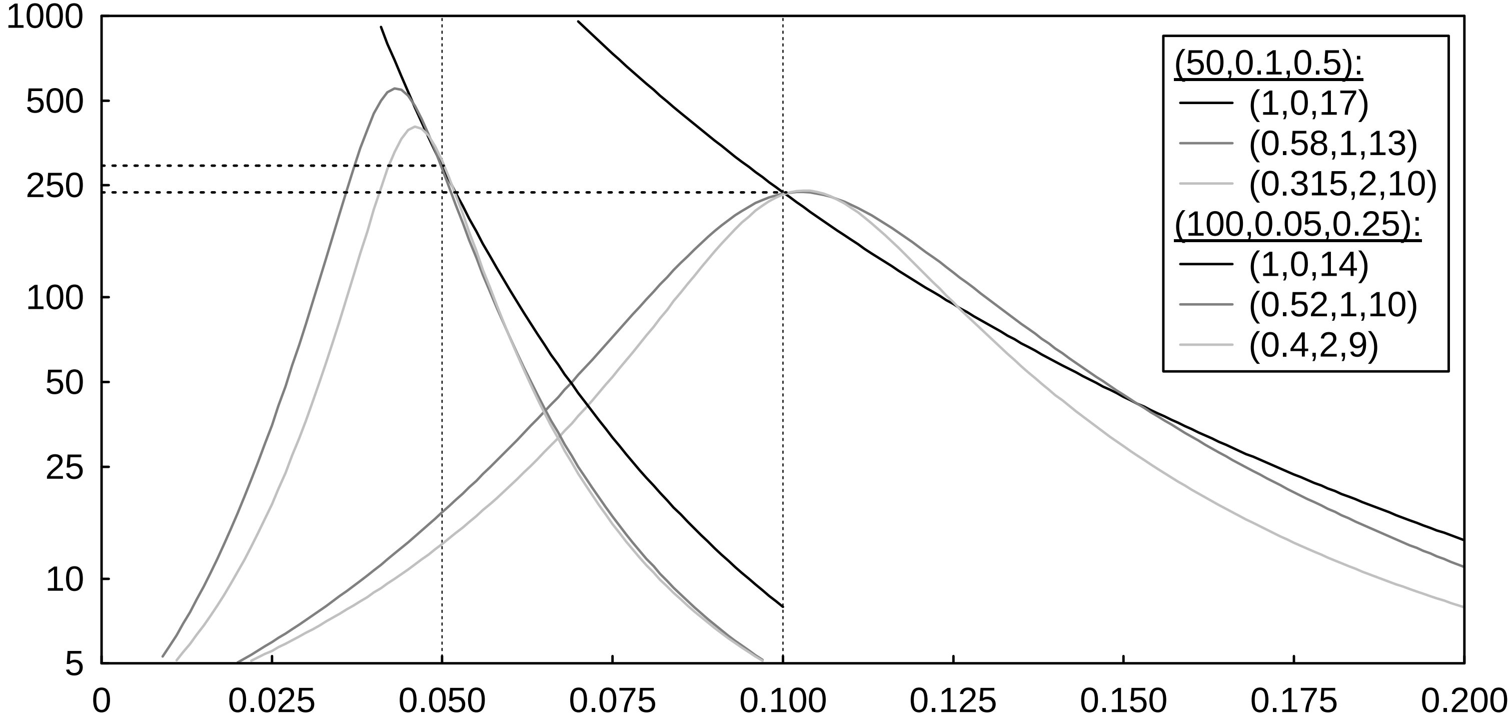
Markov EWMA Charts: Design & Performance



Out-of-control performance:

$(n, p_0, \rho_0) = (50, 0.1, 0.5)$ and $(100, 0.05, 0.25)$,

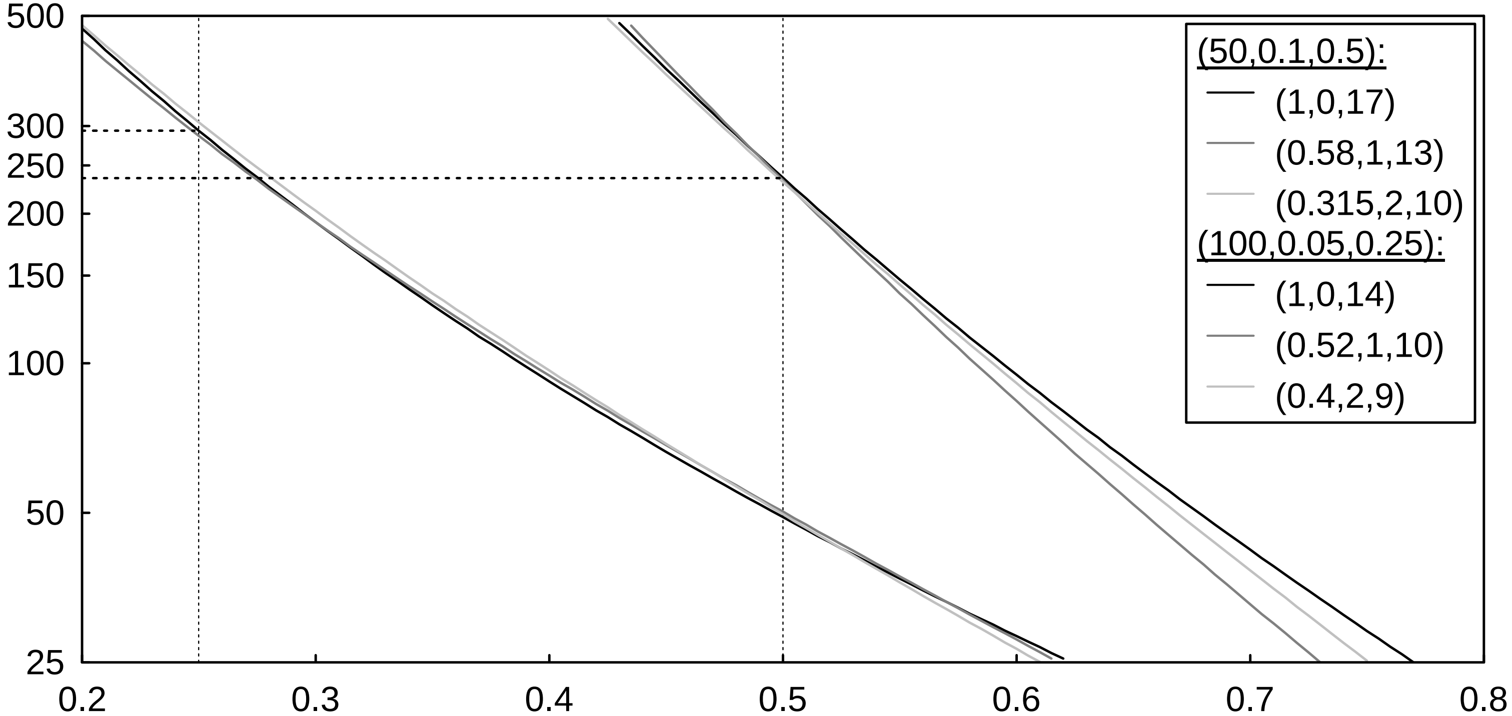
ARL_0 of about 236 and 294.



Out-of-control performance: (continued)

$(n, p_0, \rho_0) = (50, 0.1, 0.5)$ and $(100, 0.05, 0.25)$,

ARL_0 of about 236 and 294.



The
Markov Binomial
EWMA Chart

Real-data Example

High-speed attribute process: communication networks.

Statistics server accessed with high frequency, great number of different features per access $\Rightarrow$ log files from October to December 2005 required disk space $\approx$ 99.3 mb.

Complete information cannot be monitored in real-time $\Rightarrow$ confine oneself to few features measured within distant time segments.

Example: Data collected between October 17th and December 31th, 2005.

Accesses to home directory of particular member of Department of Statistics: $X_t = 1$ iff access at minute t. Segments between 9:00 and 9:59 a.m. and between 6:00 and 6:59 p.m.

$\Rightarrow$ segment sums ${C_k}^{(60)} = X_{t_k} + \ldots + X_{t_k+59}$ with $k = 1, \ldots, 110$, where each ${C_k}^{(60)}$ has range $\{0, \ldots, 60\}$.

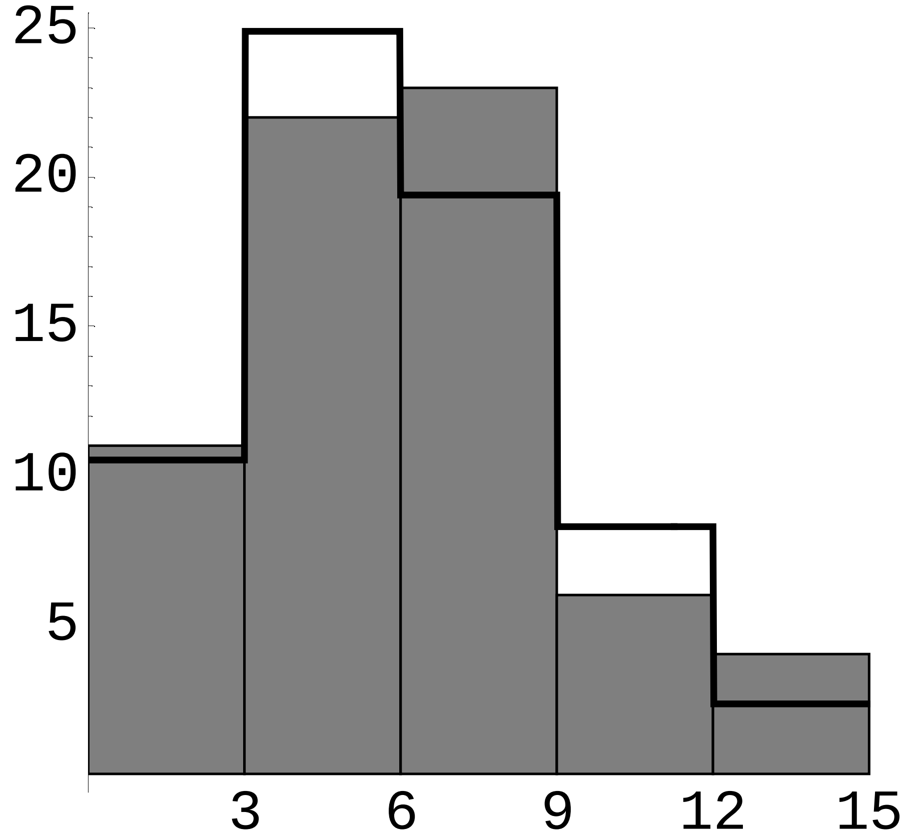
First, consider $C_1^{(60)}, \ldots, C_{66}^{(60)}$ (October 17$^{\text{th}}$ to November 30$^{\text{th}}$, 2005) $\Rightarrow$ identify in-control model!

Empirical mean and variance: 5.56061 and 9.69627

Fit Markov binomial distribution $MB(60, p, \rho)$ with $\widehat{p} \approx 0.0926768$ and $\widehat{\rho} \approx 0.320894$

Histogram with $MB(60, 0.093, 0.32)$ distribution.

Pearson's χ^2 test: p-value ≈ 0.59.



$\Rightarrow$ *In-control model*: $C_k^{(60)}$ i.i.d. $MB(60, 0.093, 0.32)$.

Markov EWMA Charts: Real-data Example

Control charts, first applied in phase I to $C_1{}^{(60)}, \ldots, C_{66}{}^{(60)}$, then in phase II to $C_{67}{}^{(60)}, \ldots, C_{110}{}^{(60)}$.

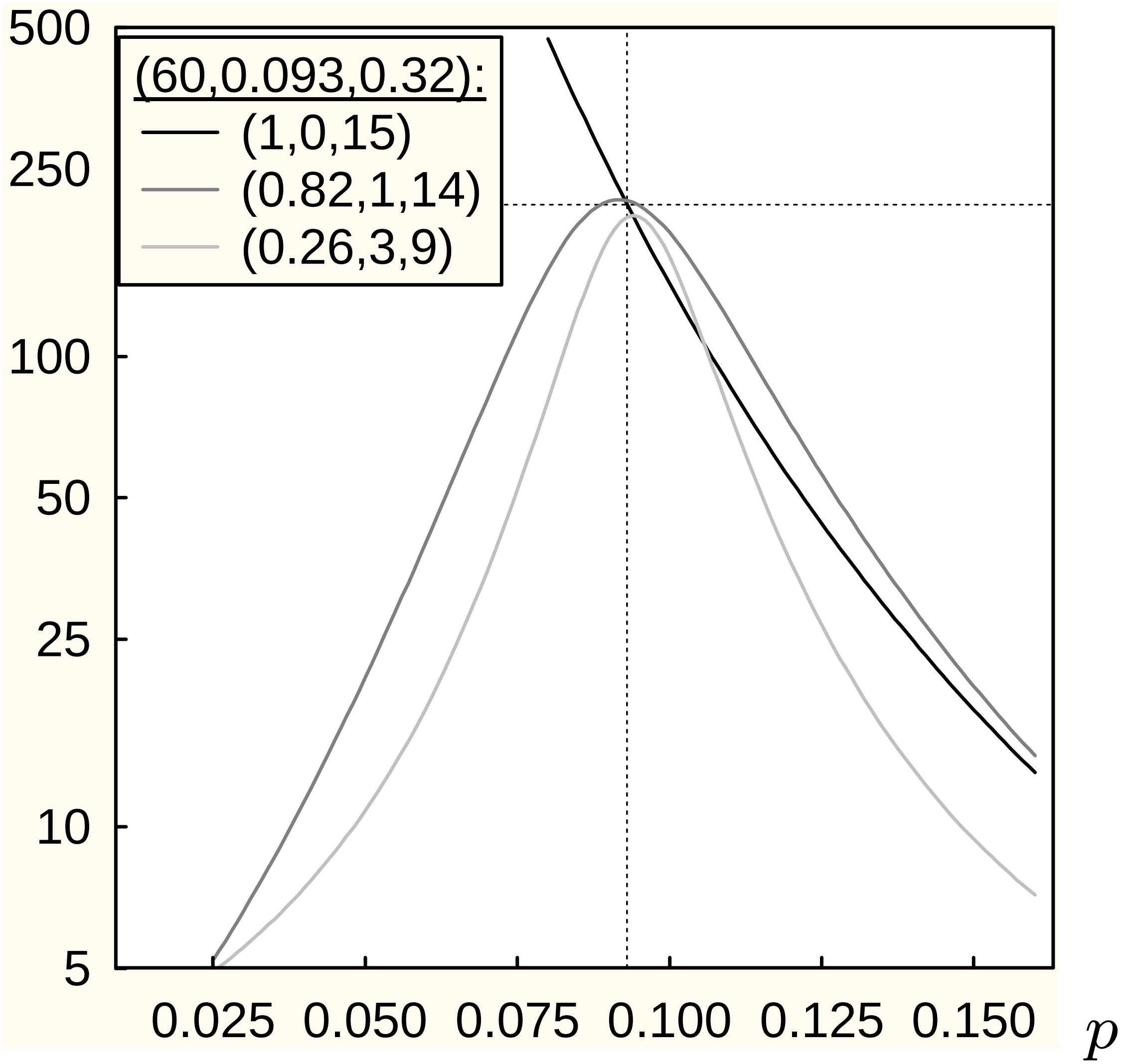
- One-sided Markov np chart with $(\lambda, l, u) = (1, 0, 15)$;
  $\rightarrow$ $ARL_0 \approx 210.634$
- Markov EWMA chart with $(\lambda, l, u) = (0.82, 1, 14)$;
  $\rightarrow$ $ARL_0 \approx 214.358$
- Markov EWMA chart with $(\lambda, l, u) = (0.26, 3, 9)$.
  $\rightarrow$ $ARL_0 \approx 197.392$

Markov EWMA Charts: Real-data Example

np chart biased,

EWMA charts

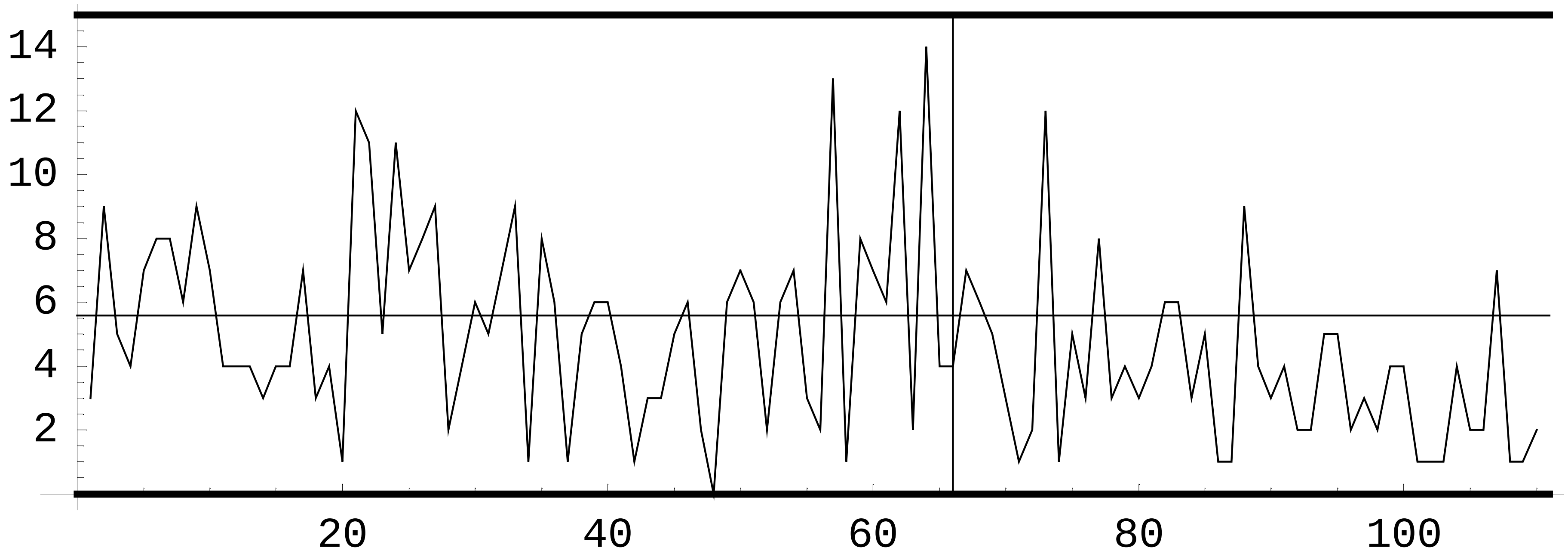
nearly unbiased.



Markov EWMA Charts: Real-data Example

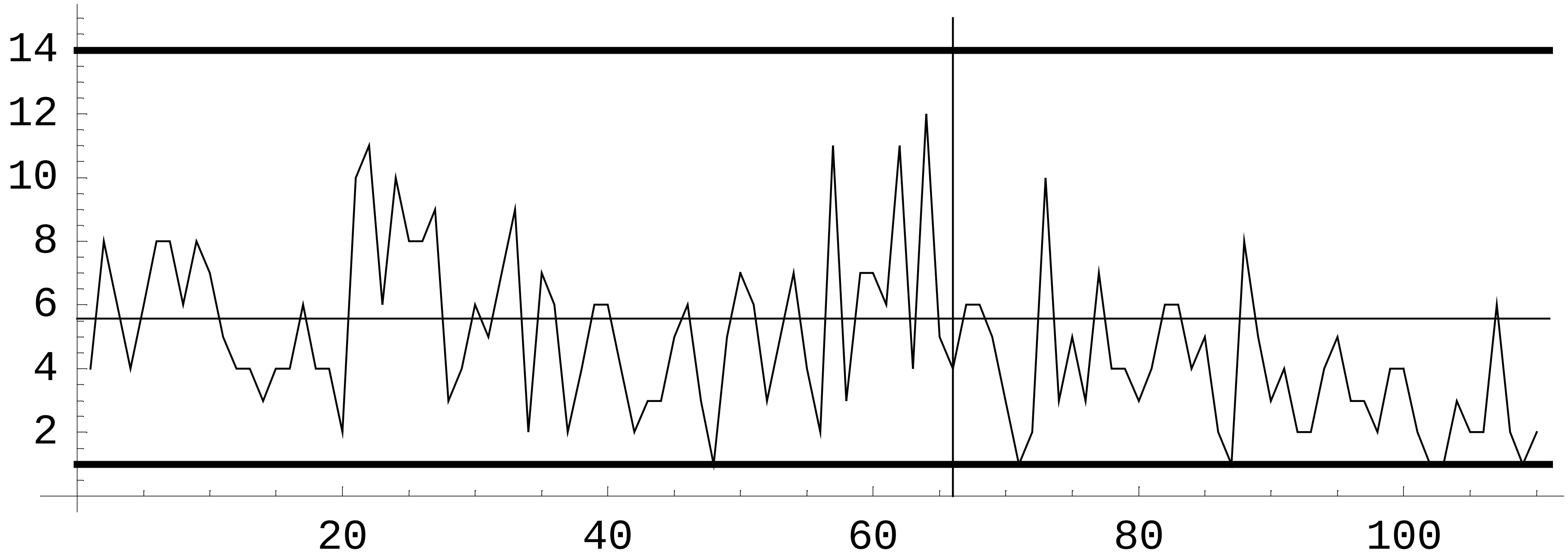
One-sided Markov np chart with $(\lambda, l, u) = (1, 0, 15)$;

$\rightarrow \; ARL_0 \approx 210.634$



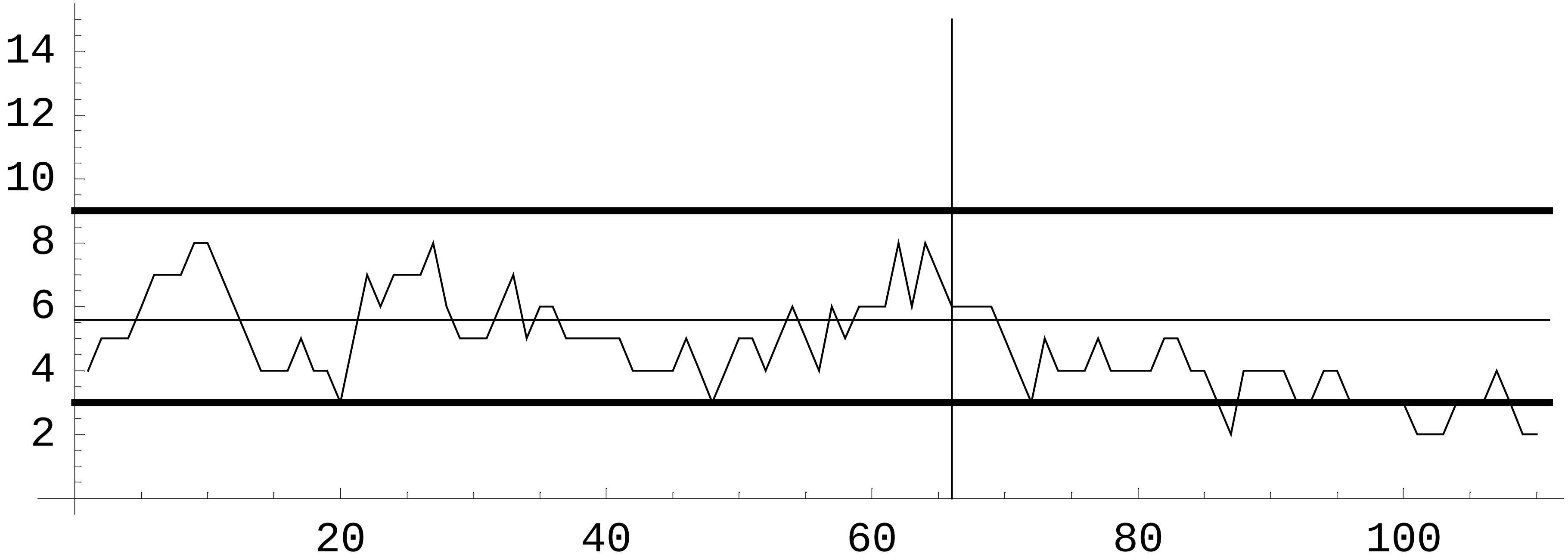
Markov EWMA chart with $(\lambda, l, u) = (0.82, 1, 14)$;

$\longrightarrow \quad ARL_0 \approx 214.358$



Markov EWMA chart with $(\lambda, l, u) = (0.26, 3, 9)$.

$\rightarrow \quad ARL_0 \approx 197.392$



Markov EWMA Charts: Real-data Example

- np chart $(1, 0, 15)$: no alarm, but decreasing trend. But several runs rules violated, e. g., $C_{89}{}^{(60)}, \ldots, C_{97}{}^{(60)}$ plotted below center line.
- EWMA chart $(0.82, 1, 14)$: no alarm, decreasing trend.
- EWMA chart $(0.26, 3, 9)$: six points below lower limit $\Rightarrow$ out-of-control.

$\Rightarrow$ Christmas season starts in December, so people busy with preparations.

Conclusions

- Monitoring distant segments from binary process;
- Markov binomial distribution to approximate distribution of the segment sums;
- Markov np chart and a Markov EWMA chart;
- exact ARL computations, performance towards several types of out-of-control situation:
  Properly designed Markov EWMA chart able to control any type of change in p and at least an increase in ρ.

Thank You
for Your Interest!

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