Comparative study of fault distinguishability based on bi- and three-valued diagnostic signals

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Abstract

The primary aim of this paper is the comparative study of fault distinguishability properties of model-based fault detection and isolation approaches. We reviewed and compared diagnostic approaches based on the diagnostic signals-faults relationship. Together, five different approaches to diagnosing were examined based on the example of a three-tank system. The paper also considers the problem of achievable fault distinguishability. The comparative study showed how three- instead of bi-valued diagnostic signals allow for increasing fault distinguishability figures. We also show that so-called column-based diagnostic reasoning might exhibit higher fault distinguishability than row-based reasoning. Finally, we formulate some practical recommendations resulting from the conducted study.

Index terms: fault isolation, bi-, and three-valued diagnostic signals, row reasoning, column reasoning, fault distinguishability

1 Introduction

The paper aims to compare fault distinguishability achievable with different inference methods applied to the same diagnosed system. The paper concerns passive diagnostic methods that do not require introducing any particular excitations [1; 2; 3; 4]. The comparative study is performed using two different approaches to diagnostic inference: column and row reasoning [5; 6; 7].

The study concerns the cases where models represent the fault-free state of the diagnosed system. We assume bi- and three-valued diagnostic signals resulting from a discrete evaluation of residuals. The way of how the residuals are evaluated is exhaustly explained in many publications, i.e., [1; 3; 5; 8]. Moreover, we assume that the quantitative knowledge of the sensitivity of residuals on faults is unknown. Therefore, the relationship between the diagnostic signals and the faults will be derived based on the expert's knowledge.

The fault inference methods referred to as column and row approaches will be analyzed for binary and three-valued diagnostic signals. In addition, all considerations will be limited to the case of single faults only. The research will not consider those methods that use, e.g., knowledge regarding the sequence of symptoms [9; 10]. A comparative study will be illustrated on an example of diagnosing a set of three serially connected tanks.

The paper contributes to the methodology of applied sciences. The contribution consists of reviewing five diagnostic inference methods based on the bi-, and three-valued relationship diagnostic signals - faults. In addition, we formulated a set of practical recommendations based on the results of conducted analysis.

The structure of the paper is as follows: Section 2 discusses the forms of notation of the fault-diagnostic signals relationship for the bi- and three-valued diagnostic signals. Section 3 describes the principles of diagnosing based on columns and rows in case of binary and three-valued residuals. The diagnosed case study and five diagnostic approaches are briefly presented in Section 4. The results of fault distinguishability study of the various diagnostic approaches are given in Section 5. Finally, Section 6 summarizes the study.

2 Fault–diagnostic signals relation

Fault isolation is a process of determining the possible faults in the diagnosed system. The fault isolation can be based on the knowledge of the values $s_j \in S$ of diagnostic signals and the mapping of diagnostic signals - faults. This mapping is determined by the Cartesian product $S \times F$ of the set of diagnostic signals

$$S = \{s_j : j = 1, 2, \ldots, J\}, \quad (1)$$

and the set of faults

$$F = \{f_k : k = 1, 2, \ldots, K\}. \quad (2)$$

This mapping is most often represented in a matrix like structure whose rows correspond to diagnostic signals and columns to faults. The form of this structure depends on the representation of diagnostic signals:

- When using a binary evaluation of absolute values of residuals, the structure has the form of the Fault Signature Matrix (FSM) [5; 6; 7]. This form of representation of fault-symptoms relationship is also referred to as: structure of residual sets [1], Boolean decision table [2], coding set [11; 12], effect of the faults on residuals [4] or binary diagnostic matrix (BDM) [13; 3].
- When a three-valued residual evaluation is used, the structure takes the form of Fault Isolation System (FIS) [3; 14].

An example of FSM is shown in Tab. 1 and FIS in Tab. 2.

Table 1: An example of a fault signature matrix.

S/F	f_1	f_2	f_3	f_4
s_1	**1**	**1**	**0**	**0**
s_2	**0**	**1**	**1**	**0**
s_3	**1**	**0**	**1**	**1**

Table 2: An example of a fault isolation system.

S/F	f_1	f_2	f_3	f_4	V_j
s_1	$\mathbf{1}$	$\mathbf{0}$	$\mathbf{1}$	$\mathbf{0}$	$\{\mathbf{0},\mathbf{1}\}$
s_2	$\mathbf{0}$	$\mathbf{-1}$	$\mathbf{-1},\mathbf{+1}$	$\mathbf{+1}$	$\{\mathbf{0},\mathbf{-1},\mathbf{+1}\}$
s_3	$\mathbf{-1}$	$\mathbf{-1},\mathbf{+1}$	$\mathbf{0}$	$\mathbf{-1}$	$\{\mathbf{0},\mathbf{-1},\mathbf{+1}\}$

The FSM and FIS differs in:

- Diagnostic signals s_j in FSM are exclusively bi-valued i.e., $V_j = \{0, 1\}$, while in FIS they can be multivalued. In addition, each diagnostic signal s_j in FIS can have its own individual set of values V_j (in which a value of 0 always indicates absence of a fault symptom). In this work we will limit our considerations only to bi- and three-valued diagnostic signals i.e., $V_j = \{0, 1\}$ or $V_j = \{0, -1, +1\}$.
- Each FSM entry corresponding to the pair $\langle s_j, f_k \rangle$ takes only one value, while any FIS entry can be a subset of the values $V_j^k \subset V_j$.
- Fault signatures in FSM corresponding to columns of Tabs. 1, 2 are referred to as simple signatures:

$$V_{FSM}(f_k) = \left[v_1^k, \dots, v_j^k, \dots, v_J^k\right]^T . \quad (3)$$

  In turn, in the case of FIS, the signatures which entries are subsets of the values of diagnostic signals are referred to as complex signatures.

$$V_{FIS}(f_k) = \left[V_1^k, \dots, V_j^k, \dots, V_J^k\right]^T . \quad (4)$$

  Diagnostic inference of k^{th} fault can be conducted based on the analysis of the degree of conformity of fault-specific signature with the vector of the current values of diagnostic signals (observations). If the principle of complete compliance is adopted, the reasoning of faults may take the form of rules (5, 6). For example, in the FSM case, the rules for faults are:

$$if\left(s_1 = v_1^k\right) \dots \wedge \dots \left(s_J = v_J^k\right) then\, f_k, \quad (5)$$

  while in the case of FIS, the rules are as follows:

$$if\left(s_1 \in V_1^k\right) \dots \wedge \dots \left(s_J \in V_J^k\right) then\, f_k, \quad (6)$$

- In the FSM, each j^{th} row of the matrix is a transposed vector of the faults to which the diagnostic signal s_j is sensitive. The diagnostic inference determines a subset of potential faults if a diagnostic signal of fault takes the value of 1, i.e., $s_j = 1$. This method of inference can be presented in the form of a set of J rules:

$$if(s_j = 1)\, then\, f_j \in F(s_j = 1), \forall j \in \{1..J\} . \quad (7)$$

  Symbol $F(s_j = 1)$ denotes the set of faults to which the signal s_j is sensitive. In the case of FIS, by three-valued diagnostic signal values, the diagnosis consists of two subsets of faults, associated with any non-zero value of the fault symptom. In this case, the inference rules are:

$$if(s_j = -1)\, then\, f_j \in F(s_j = -1), \quad (8)$$

$$if(s_j = +1)\, then\, f_j \in F(s_j = +1), \quad (9)$$

  where: $F(s_j = -1)$ and $F(s_j = +1)$ are subsets of faults that are indicated by a symptom $s_j = -1$ or $s_j = +1$ respectively.

All other forms of the relationship between binary or multivalued diagnostic signals and faults are secondary to the FSM or FIS. Therefore, diagnostic inference methods, i.e., based on if-then rules and logical implications or fault trees, can be derived from FSM or FIS [9].

3 Diagnosing with columns and rows

The two principally different approaches of diagnosing can be distinguished:

- Column Reasoning, also referred to as signature-based inference [1; 5; 7], or parallel inference [3];
- Row Reasoning, also referred to as symptom based inference or serial inference [3].

Columns or rows refer to a tabular form of notion of the relationship diagnostic signals - faults (see Tabs. 1, 2). Knowledge of this relationship is necessary for the isolation of faults. Column-based and row-based diagnostic inference approaches can be applied both for binary and three-valued diagnostic signals.

The practical usability of these approaches is because they are based on the models representing the fault-free state of the diagnosed system. It is unnecessary to know the quantitative impact of faults on residuals, i.e., it is unnecessary to know residuals in the inner form (Gertler, 1998). In the FSM, only the knowledge of the sensitivity of diagnostic signals to faults is necessary. Optionally, the FIS makes use of the knowledge regarding the sign of the diagnostic signal too.

The column-based reasoning of faults is commonly used in the FDI community, while row-based diagnostic inference approaches within the DX community. There are also known works [13; 15] in which row-based inference approaches are used that are different from these proposed by Reiter [16].

Derivative and mixed approaches based on diagnostic matrices referred to as dynamic, such as $MUFIA$ [17], and MFI [18] are also known. As shown in a comparative studies [18; 19], MFI and $MUFIA$ have a superior computational effectiveness.

Bartyś in [20] introduced the alternative and dominant signatures that allow generalizing the rule-based inference of faults. The reasoning scheme was presented in equivalent conditional statements referring to the introduced concepts of alternative, dominant and mixed premises.

In this study, we analyze the properties of two classes of diagnostic inference by the assumption of single fault occurrence. We also assume that all symptoms of faults persist throughout the fault isolation process [5; 6; 21].

In the case of column-based inference, a zero value of diagnostic signal value means that none of the faults occurs to which this particular signal is sensitive [13]. This interpretation is known as exoneration assumption [5]. We also assume that all symptoms of a fault must occur (completeness of symptoms). Column-based inference indicates faults whose signatures match vectors of current diagnostic signal values.

In the case of row-based reasoning, we assume that the appearance of a symptom (a non-zero diagnostic signal value) indicates the fault to which the diagnostic signal is sensitive. Thus, when assuming single faults, row-based inference results in pointing out all subsets of faults $F(s_j \neq 0)\ \forall j \in \{1..J\}$. All row- and column-based reasoning assumptions are thoroughly discussed in [5; 7].

In this paper we discuss the fault distinguishability obtainable by the five following inference methods:

a) CB (Column-Binary) - based on columns and binary diagnostic signals. This approach is commonly used in the FDI community. Examples of CB are given in Gertler's works, e.g., [1]. Signatures of the shape as in (3) are used in the rule-based diagnostic inference according to rules (5). The diagnosis takes the form (10).

$$DGN_{CB} = \left\{ f_k : \vee_j (s_j = v_j^k) \right\}. \tag{10}$$

b) CT (Column-Three-valued) is the inference based on columns and three-valued diagnostic signals. The inference algorithm was studied in [3]. Here, the inference of faults is based on complex signatures (4) corresponding to the FIS columns. The inference rules have the form as in (6).

$$DGN_{CT} = \left\{ f_k : \vee_j (s_j \in V_j^k) \right\}. \tag{11}$$

c) RB (Row-Binary) is FSM row-based reasoning with binary diagnostic signals. This approach is referred to as symptom-based reasoning [22]. Rules (7) are used for inference. The diagnosis indicates a set of faults that is a product of all subsets $F(s_j = 1)$ for which symptoms of faults have occurred.

$$DGN_{RB} = \{ f_k : \cap F_j(s_j = 1) \}. \tag{12}$$

d) RT (Row-Three-valued) is row-based reasoning in FIS with three-valued diagnostic signals. The rules (8) and (9) corresponding to the rows of the FIS are used for inferring.

$$DGN_{RT} = \{ f_k : \cap F(s_j \neq 0) \} \tag{13}$$

e) DTS is the row-based inference in FSM with binary diagnostic signals without exoneration assumption. The approach was presented in [13]. The diagnosis is performed only after the values of all diagnostic signals $s_j \in S$ settle. Its form is as follows:

$$DGN_{DTS} = \{ f_k : \cap F(s_j = 1) \backslash \cap F(s_j = 0) \}. \tag{14}$$

All of the above approaches are also usable in the case of inference assuming single and double faults. In this paper, the study will be limited only to single faults. It is one of the reasons why the DX methods [16; 23; 24; 25] are not considered in this study. DX diagnoses are hitting sets of all minimal conflict sets that are observed. They indicate single and multiple faults. It is a significant advantage of this approach. By reducing the considered fault multiplicity in potential diagnoses, the DX reasoning indicates faults of all conflict sets. It is equivalent to diagnoses DGN_{RB} (12).

4 Case study

A comparison of the fault distinguishability with the use of the approaches of diagnostic inference characterized in Sec. 3 will be carried out on the example of diagnosing a set of three serially connected buffer tanks. A control valve throttles the liquid inflow supplying the first tank. The third tank provides free liquid outflow (Fig. 1). The systems of serially connected tanks are commonly used to exemplify developed diagnostic approaches of dynamic systems due to the ease of understanding the principles of their operation and a sufficient degree of complexity resulting from the feedback loops in the diagnosed system. [26; 27; 28; 29].

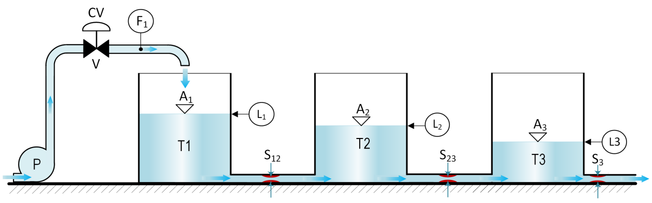

Figure 1: Three-tank system

Tab. 3 presents a specification of measurement and control signals used for diagnostic purposes, while Tab. 4 provides a list of considered faults.

Table 3: The set of process variables.

$Process\ value$	Tag
$Valve\ control\ signal$	CV
$Inflow\ rate\ of\ the\ liquid\ into\ the\ tank\ T_1$	F_1
$Level\ of\ the\ liquid\ in\ the\ tank\ T_1$	L_1
$Level\ of\ the\ liquid\ in\ the\ tank\ T_2$	L_2
$Level\ of\ the\ liquid\ in\ the\ tank\ T_3$	L_3

Table 4: The set of considered faults.

F	Tag
f_1	$sensor\ F_1\ fault$
f_2	$sensor\ L_1\ fault$
f_3	$sensor\ L_2\ fault$
f_4	$sensor\ L_3\ fault$
f_5	$the\ fault\ in\ the\ physical\ link\ of\ CV$
f_6	$control\ valve\ fault$
f_7	$pump\ fault\ (change\ in\ outflow\ rate)$
f_8	$dry-run\ of\ the\ pump$
f_9	$deposits\ in\ the\ pipe\ connecting\ T_1\ and\ T_2$
f_{10}	$deposits\ in\ the\ pipe\ connecting\ T_2\ and\ T_3$
f_{11}	$deposits\ in\ outlet\ pipe\ in\ tank\ T_3$
f_{12}	$leaky\ tank\ T_1$
f_{13}	$leaky\ tank\ T_2$
f_{14}	$leaky\ tank\ T_3$

Let us assume that the four nonlinear phenomenological partial system models will be used for fault detection. The models of the final control element and fluid flow balance for the set of tanks are depicted in Tab. 5. Residuals, and then bi- and three-valued diagnostic signals, are generated based on these models.

Table 5: The diagnostic tests.

S	$Residual\ equations$
s_1	$r_1 = F_1 - \hat{F} = F_1 - f(CV)$
s_2	$r_2 = F_1 - \alpha_{12} S_{12} \sqrt{2g(L_1 - L_2)} - A_1 \frac{dL_1}{dt}$
s_3	$r_3 = \alpha_{12} S_{12} \sqrt{2g(L_1 - L_2)} +$ $-\alpha_{23} S_{23} \sqrt{2g(L_2 - L_3)} - A_2 \frac{dL_2}{dt}$
s_4	$r_4 = \alpha_{23} S_{23} \sqrt{2g(L_2 - L_3)}$ $-\alpha_3 S_3 \sqrt{2gL_3} - A_3 \frac{dL_3}{dt}$
$Notion: \alpha_{ij} - flow\ contraction\ coefficient$ $between\ i^{th} and\ j^{th}\ tank; S_{ij} - cross-sectional$ $area\ of\ pipes\ connecting\ i^{th}\ and\ j^{th}\ tanks;$ $A_i - cross-sectional\ area\ of\ the\ i^{th} tank.$	

Tabs. 6 and 7 show respectively the FSM and FIS for the set of tanks. Both matrices were filled in based on expert knowledge. For the transparency of presentation, the zero values were omitted.

Table 6: FSM for four binary diagnostic signals.

S/F	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}	f_{11}	f_{12}	f_{13}	f_{14}
s_1	1				1	1	1	1						
s_2	1	1	1						1			1		
s_3		1	1	1					1	1			1	
s_4			1	1						1	1			1

Table 7: FIS for four three-valued diagnostic signals.

S/F	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}	f_{11}	f_{12}	f_{13}	f_{14}
s_1	$+1$ -1				$+1$ -1	$+1$ -1	-1	-1						
s_2	$+1$ -1	$+1$ -1	$+1$ -1						$+1$			-1		
s_3		$+1$ -1	$+1$ -1	$+1$ -1					-1	$+1$			-1	
s_4			$+1$ -1	$+1$ -1						-1	$+1$			-1

5 Study of fault distinguishability

In this study, we consider all approaches described in Section 3. As a quantitative measure of fault distinguishability, we introduce the index of theoretical diagnostic accuracy. We define the theoretical accuracy of a single diagnosis as the reciprocal of the number of faults d_i indicated in the diagnosis. The diagnostic accuracy D is the average accuracy of single fault diagnoses:

$$D = \frac{1}{K} \sum_{k=1}^{K} \frac{1}{d_k}. \tag{15}$$

In the case of faults for which different diagnoses can be obtained depending on the observed diagnostic signal values (this is typical by three-valued diagnostic signals), the accuracy of diagnosing is the average accuracy of all physically plausible diagnoses obtained for all diagnostic signals. Diagnoses are determined for all possible faults and all fault isolation approaches considered in the paper. The uncertainties of diagnostic signals, modeling errors, disturbances, and measurement noises are neglected.

The obtained results of the diagnostic accuracy indices are listed in Tab. 8. In turn, the diagnoses obtained with the studied approaches are shown in Appendix A1.

Table 8: The fault isolation accuracies obtained for the three-tank system.

	CB	RB	DTS	CT	RT
D	0.571	0.352	0.571	0.696	0.408

Observation 1. Tab. 8 shows that the column-based reasoning of faults exhibits higher fault distinguishability than row-based reasoning for the considered case study. This observation applies both to diagnosing based on binary and three-valued signals. The exoneration assumption plays here a primary role. It can be clearly seen, for example, in case of the DTS approach, where row-based inference is applied without exoneration of symptoms. In this particular case, the value of the accuracy index of diagnosis is equal to this obtained in the column-based CB approach.

Observation 2. The second observation resulting from the conducted study concerns the significant increase in the value of the accuracy index D by three-valued diagnostic signals compared with binary signals. This observation confirms the expectations and results of previous studies [9; 27]. Clearly, the additional knowledge, if adequate, improves the quality of fault isolation.

6 Final remarks

This paper aims to compare the distinguishability of faults obtained by inference based on five different approaches to diagnosing a three-tank system. It was shown in the example that using three-valued diagnostic signals could improve the accuracy of the diagnoses. Based on this observation, we recommend using the FIS instead of FSM to represent the fault-diagnostic signals relationship anywhere it is possible. Generally, the column-based inference provides higher distinguishability over row-based inference. However, in case of column-based reasoning, the false diagnoses generated by transients of diagnostic signals should be taken into consideration. In turn, the row-based inference is devoided of this disadvantage.

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Appendix A1

Table 9: List of diagnoses obtained by various approaches.

f_k	$s_1,..,s_4$ *binary*	CB	RB (DX)	DTS	CT	RT	$s_1,..,s_4$ *three − val.*
f_1	$1,1,0,0$	$\{f_1\}$	$\{f_1\}$	$\{f_1\}$	$\{f_1\}$	$\{f_1\}$	$+1,+1,\ 0,\ 0$
					$\{f_1\}$	$\{f_1\}$	$-1,-1,\ 0,\ 0$
f_2	$0,1,1,0$	$\{f_2,f_9\}$	$\{f_2,f_3,f_9\}$	$\{f_2,f_9\}$	$\{f_2\}$	$\{f_2,f_3\}$	$0,-1,+1,\ 0$
					$\{f_2,f_9\}$	$\{f_2,f_3,f_9\}$	$0,+1,-1,\ 0$
f_3	$0,1,1,1$	$\{f_3\}$	$\{f_3\}$	$\{f_3\}$	$\{f_3\}$	$\{f_3\}$	$0,+1,-1,+1$
					$\{f_3\}$	$\{f_3\}$	$0,-1,+1,-1$
f_4	$0,0,1,1$	$\{f_4,f_{10}\}$	$\{f_3,f_4,f_{10}\}$	$\{f_4,f_{10}\}$	$\{f_4,f_{10}\}$	$\{f_3,f_4,f_{10}\}$	$0,\ 0,+1,-1$
					$\{f_4\}$	$\{f_4\}$	$0,\ 0,-1,+1$
f_5	$1,0,0,0$	$\{f_5,f_6,f_7,f_8\}$	$\{f_1,f_5,f_6,f_7,f_8\}$	$\{f_5,f_6,f_7,f_8\}$	$\{f_5,f_6\}$	$\{f_1,f_5,f_6\}$	$+1,\ 0,\ 0,\ 0$
					$\{f_5,f_6,f_7,f_8\}$	$\{f_1,f_5,f_6,f_7,f_8\}$	$-1,\ 0,\ 0,\ 0$
f_6	$1,0,0,0$	$\{f_5,f_6,f_7,f_8\}$	$\{f_1,f_5,f_6,f_7,f_8\}$	$\{f_5,f_6,f_7,f_8\}$	$\{f_5,f_6\}$	$\{f_1,f_5,f_6\}$	$+1,\ 0,\ 0,\ 0$
					$\{f_5,f_6,f_7,f_8\}$	$\{f_1,f_5,f_6,f_7,f_8\}$	$-1,\ 0,\ 0,\ 0$
f_7	$1,0,0,0$	$\{f_5,f_6,f_7,f_8\}$	$\{f_1,f_5,f_6,f_7,f_8\}$	$\{f_5,f_6,f_7,f_8\}$	$\{f_5,f_6,f_7,f_8\}$	$\{f_1,f_5,f_6,f_7,f_8\}$	$-1,\ 0,\ 0,\ 0$
f_8	$1,0,0,0$	$\{f_5,f_6,f_7,f_8\}$	$\{f_1,f_5,f_6,f_7,f_8\}$	$\{f_5,f_6,f_7,f_8\}$	$\{f_5,f_6,f_7,f_8\}$	$\{f_1,f_5,f_6,f_7,f_8\}$	$-1,\ 0,\ 0,\ 0$
f_9	$0,1,1,0$	$\{f_2,f_9\}$	$\{f_2,f_3,f_9\}$	$\{f_2,f_9\}$	$\{f_2,f_9\}$	$\{f_2,f_3,f_9\}$	$0,+1,-1,\ 0$
f_{10}	$0,0,1,1$	$\{f_4,f_{10}\}$	$\{f_3,f_4,f_{10}\}$	$\{f_4,f_{10}\}$	$\{f_4,f_{10}\}$	$\{f_3,f_4,f_{10}\}$	$0,\ 0,+1,-1$
f_{11}	$0,0,0,1$	$\{f_{11},f_{14}\}$	$\{f_3,f_4,f_{10},f_{11},f_{14}\}$	$\{f_{11},f_{14}\}$	$\{f_{11}\}$	$\{f_3,f_4,f_{11}\}$	$0,\ 0,\ 0,+1$
f_{12}	$0,1,0,0$	$\{f_{12}\}$	$\{f_1,f_2,f_3,f_9,f_{12}\}$	$\{f_{12}\}$	$\{f_{12}\}$	$\{f_1,f_2,f_3,f_{12}\}$	$0,-1,\ 0,\ 0$
f_{13}	$0,0,1,0$	$\{f_{13}\}$	$\{f_2,f_3,f_4,f_9,f_{13}\}$	$\{f_{13}\}$	$\{f_{13}\}$	$\{f_2,f_3,f_4,f_9,f_{13}\}$	$0,\ 0,-1,\ 0$
f_{14}	$0,0,0,1$	$\{f_{11},f_{14}\}$	$\{f_3,f_4,f_{10},f_{11},f_{14}\}$	$\{f_{11},f_{14}\}$	$\{f_{14}\}$	$\{f_3,f_4,f_{10},f_{14}\}$	$0,\ 0,\ 0,-1$