Communication using the discrete Nonlinear Fourier Spectrum:
Counter-propagating Raman and EDFA

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Outline

• Motivation

• Introduction to the Nonlinear Fourier Transform

• Modulation of discrete spectrum

• Transmission in EDFA / counter-propagating Raman amplifier links

• Conclusion / prospect
Motivation

- Modern optical transmission are using advanced modulation formats and coherent detection
- Current transmission formats are designed for linear channels
  ⇒ Fiber is a nonlinear medium due to Kerr nonlinearity
- Novel transmission formats to consider fiber nonlinearity
  ⇒ Theoretical framework of Nonlinear Fourier Transformation / Inverse Scattering Theory assumes lossless transmission or ideal distributed amplification
- Theoretical assumptions are hard to meet with current commercially available equipment and installed fibers
Signal Propagation in an Optical Fiber

- Field propagation in an optical fiber: Nonlinear Schrödinger Equation
- "Linearization" of the nonlinear channel

\[ i \frac{\partial A(t,z)}{\partial z} - \frac{\beta_2}{2} \frac{\partial^2 A(t,z)}{\partial t^2} + \gamma |A(t,z)|^2 A(t,z) = 0 \]

\[ H(\lambda_j, L) = \exp(-i2\lambda_j^2 L) \]
Basic Idea

- We look for an invariant under evolution (noise and loss neglected)

- Normalization of NLSE \(\Rightarrow\) transition to "soliton"-units

\[
i \frac{\partial q(t, z)}{\partial z} + \frac{1}{2} \frac{\partial^2 q(t, z)}{\partial t^2} + |q(t, z)|^2 q(t, z) = 0
\]

- Normalization constants

\[
q = A \sqrt{\gamma} Z_s, \quad z = \frac{Z}{Z_s}, \quad t = \frac{T}{T_s}, \quad Z_s = \frac{T_s^2}{|\beta_2|}
\]

\(\gamma\): nonlinear coefficient, \(\beta_2\): group velocity dispersion coefficient, \(T_s\): free time scale
Basic Idea

- We look for an invariant under evolution (noise and loss neglected)
- Normalization of NLSE $\Rightarrow$ transition to "soliton"-units

$$i \frac{\partial q(t, z)}{\partial z} + \frac{1}{2} \frac{\partial^2 q(t, z)}{\partial t^2} + |q(t, z)|^2 q(t, z) = 0$$

- Nonlinear Fourier spectrum

Continuous: $\hat{q}(\lambda) = \frac{b(\lambda)}{a(\lambda)} \quad (\lambda \in \mathbb{R})$

Discrete: $\lambda_j \quad (a(\lambda_j) = 0, \lambda \in \mathbb{C}, \Im(\lambda_j) > 0,$

$$\tilde{q}(\lambda_j) = \frac{b(\lambda_j)}{\frac{d a(\lambda)}{d \lambda}|_{\lambda=\lambda_j}} \quad j = 1, \ldots, N)$$
Modulation of discrete spectrum

- Fundamental soliton:

\[ q(t) = 2 \mathcal{I}(\lambda_j) \text{sech}(2 \mathcal{I}(\lambda_j) \cdot (t - t_0)) \cdot e^{-i \mathcal{K}(\lambda_j) t} \cdot e^{-i \Phi_s} \]

- Degrees of freedom: \( \mathcal{I}(\lambda_j), \mathcal{K}(\lambda_j), t_0, \Phi_s \)

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Modulation of discrete spectrum

- Fundamental soliton:
  \[ q(t) = 2I(\lambda_j) \operatorname{sech} (2I(\lambda_j) \cdot (t - t_0)) \cdot e^{-i\mathcal{H}(\lambda_j)t} \cdot e^{-i\Phi_s} \]

- Degrees of freedom: \( I(\lambda_j), \mathcal{R}(\lambda_j), t_0, \Phi_s \)

- Evolution of the pulse center depending on eigenvalue and position\(^1\)
  \[ t_0(z) = 4\mathcal{R}(\lambda_j) z + \frac{1}{2I(\lambda_j)} \log \left( \frac{|\tilde{q}_j|}{2\tilde{I}(\lambda_j)} \right) \]

- Utilization of precise algorithm to control \( \tilde{q}(\lambda_j) \)\(^2\)

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\(^1\) Gui et al. “High-order modulation on a single discrete eigenvalue for optical communications based on nonlinear Fourier transform”. 2017.

Modulation of discrete spectrum

Fiber: NZ-DSF, Symbolrate: 1GBd, $T_s = \frac{T_{\text{Sym}}}{22}$, $Z_s = 359.3\text{km}$

$\mathcal{I}(\lambda_j) = -10^{-5}0510$

$\mathcal{R}(\lambda_j)$

$Z = -1508\text{km}$

$Z = 0\text{km}$

$Z = 1508\text{km}$

$\mathcal{I}(\tilde{q}) = -10^{-5}0510$

$\mathcal{R}(\tilde{q})$
\[
\alpha = 0.226 \text{dB/km } (\text{span loss} > 17 \text{dB}) \quad \beta_2 = -5.75 \text{ps}^2/\text{km} \quad \gamma = 1.6 \text{W/km}
\]
Consideration of Fiber Losses

- Theoretical assumption of lossless fiber for integrable differential equation collides with realistic link

- Periodic amplification by EDFA’s after fiber span vs. distributed or ideal distributed Raman amplification

- Consideration of power variation by averaging over a virtual lossless link with reduced nonlinearity (LPA$^3$)

\[
k = \frac{\int_0^{L_{\text{span}}} \exp \left( 2 \int_0^z g(y) dy \right) dz}{L_{\text{span}}} \]

\[
\gamma_{\text{eff}} = \gamma \cdot k, \quad \bar{P} = \frac{\int_{-T_{\text{Sym}}/2}^{T_{\text{Sym}}/2} \frac{|q(T)|^2}{\sqrt{\gamma_{\text{eff}} Z_s}} dT}{T_{\text{Sym}}} \]

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3 Le et al. “Modified nonlinear inverse synthesis for optical links with distributed Raman amplification”. 2015.
- Lumped amplification:
  \[ k = 0.237, \quad \bar{P} = -3.97\text{dBm} \]
Counter-Raman & EDFA amplification

- Counter-propagating Raman - $P_1 = 320\text{mW}$, $P_2 = 220\text{mW}$:
  \[ k = 0.388, \bar{P} = -6.12\text{dBm} \]

![Graph showing power variation in dB against distance (Z) in km.](image)
Counter-Raman & EDFA amplification

- Counter-propagating Raman - $P_1 = 320$ mW, $P_2 = 280$ mW:
  $k = 0.474, \bar{P} = -7.00$ dBm
EDFA amplification

- Lumped amplification: \( k = 0.237, \bar{P} = -3.97\, \text{dBm} \)

![Graph showing BER and Z vs. B2B and Z values]
EDFA amplification

- Lumped amplification: \( k = 0.237, \bar{P} = -3.97\,\text{dBm} \)

\[ I(\lambda_j) \]

\[ 3016\,\text{km} \]

BER

\[ P_{\text{input}} \, [\text{dBm}] \]

\[ 3016\,\text{km} \]

\[ 3016\,\text{km} \]
EDFA amplification

- Lumped amplification: $k = 0.237, \bar{P} = -3.97\,\text{dBm}$
Counter-propagating Raman amplification

- Counter Raman: $k = 0.388$, $\bar{P} = -6.11\,\text{dBm}$
Counter-propagating Raman amplification

- Counter Raman: \( k = 0.474, \bar{P} = -7.00\text{dBm} \)

![Graph showing BER vs. input power for various distances]
Counter-propagating Raman amplification

- Counter Raman: $k = 0.388$, $\bar{P} = -6.11\,\text{dBm}$, $Z = 2262\,\text{km}$
Transmission results

- Slow / simple signal: reliable EDFA based transmission (BER: $6.8 \times 10^{-5}$ @ 3016 km with LMMSE equalization of $b(\lambda_j)$)

- Moderate Raman amplification (compensating span loss):
  - Reduction of eigenvalue and spectral amplitude variance by a factor 2..5 (BER: $<3 \times 10^{-6}$ @ 3016 km)

- Intense Raman amplification (overcompensating span loss):
  - Reduction of eigenvalue variance by a factor 2, no effect on spectral amplitude (BER: $\approx 3 \times 10^{-6}$ @ 3016 km)

(Within a propagation distance of 3450 km the fastest eigenvalue travels through one symbol $T_{Sym} = 1$ ns (99.9% energy))
Conclusion / prospect

- First stokes order counter-propagating Raman amplification is suitable for NFT transmission

- First stokes order co-propagating Raman amplification can hardly be realized with commercial pump modules (FBG stabilized FP laser) due to high RIN penalty

- Second stokes order Raman amplification is not yet commercially available on a system level, implementing FBG in existing fibers would be challenging

⇒ Investigation of influence of amplification period for EDFA and Raman amplified signals

⇒ Investigation of interaction of multiple eigenvalues
References


T. Gui, C. Lu et al., "High-order modulation on a single discrete eigenvalue for optical communications based on nonlinear Fourier transform", Opt Express, vol. 25, no. 17, pp.20286-20297, 08/2017

S.T. Le, V. Aref, H. Buelow, "Nonlinear signal multiplexing for communication beyond the Kerr nonlinearity limit", Nat Photonics, vol. 11, no. 9, pp. 570-576, 09/2017

Thank you for your attention!

Questions?

Counter-propagating Raman amplification

- Counter Raman: $k = 0.388$, $\bar{P} = -6.11$ dBm, $Z = 2262$ km

![Graph showing the relationship between input power ($P_{\text{input}}$) and variance of the discrete nonlinear Fourier spectrum. The graph includes symbols for $I(\lambda_j)$, $R(\lambda_j)$, $|\hat{q}(\lambda_j)|$, and $\arg(\hat{q}(\lambda_j))$.](image-url)
Counter-propagating Raman amplification

- Counter Raman: $k = 0.388$, $\bar{P} = -6.11\text{dBm}$, $Z = 3016\text{km}$
Counter-propagating Raman amplification

- Counter Raman: $k = 0.474$, $\bar{P} = -7.00\, \text{dBm}$, $Z = 2262\, \text{km}$
Counter-propagating Raman amplification

- Counter Raman: \( k = 0.474, \bar{P} = -7.00 \text{dBm}, Z = 3016 \text{km} \)