Diskussionspapierreihe
Working Paper Series

Helmut Schmidt Universität
Universität der Bundeswehr Hamburg

The Boulding-Richardson Model Revisited

Klaus Beckmann
Susan Gattke
Lennart Reimer

Nr./ No. 159
June 2015

Department of Economics
Fächergruppe Volkswirtschaftslehre

Autoren / Authors

Klaus Beckmann
Helmut-Schmidt-University Hamburg
Chair for Public Economics
Holstenhofweg 85, 22043 Hamburg
klaus.beckmann@hsu-hh.de

Susan Gattke
Helmut-Schmidt-University Hamburg
Chair for Public Economics
Holstenhofweg 85, 22043 Hamburg
gattke@hsu-hh.de

Lennart Reimer
Helmut-Schmidt-University Hamburg
Chair for Public Economics
Holstenhofweg 85, 22043 Hamburg
lennart.reimer@hsu-hh.de

Redaktion / Editors
Helmut Schmidt Universität Hamburg / Helmut Schmidt University Hamburg
Fächergruppe Volkswirtschaftslehre / Department of Economics

Eine elektronische Version des Diskussionspapiers ist auf folgender Internetseite zu finden / An electronic version of the paper may be downloaded from the homepage:

http://fgvwl.hsu-hh.de/wp-vwl

Koordinator / Coordinator
Klaus B. Beckmann
wp-vwl@hsu-hh.de

Helmut Schmidt Universität Hamburg / Helmut Schmidt University Hamburg
Fächergruppe Volkswirtschaftslehre / Department of Economics
Diskussionspapier Nr. 159
Working Paper No. 159

The Boulding-Richardson Model Revisited

Klaus Beckmann

Susan Gattke

Lennart Reimer

Zusammenfassung / Abstract

We review, and extend, one of the classic dynamic models of conflict in economics by Richardson (1919) and Boulding (1962). It turns out that the stability properties of the model change if one takes a more realistic "incrementalist" view, and that chance / friction can easily be incorporated into the standard model by defining a probability of (de-)escalation. An application of the model to GDELT data on the Ethiopian-Eritreian war (1998-2000) reveals some problems with the psychological interpretation of the Richardson equations.

JEL-Klassifikation / JEL-Classification: B25; D74

Schlagworte / Keywords: Conflict dynamics, psychology of aggression, escalation, stability, patterns of conflict.

1. Introduction

The formal, quantitative analysis of dynamic conflict in economics began in 1916, when Frederick Lanchester (1956) developed his linear and square laws of attrition. Such models consist of systems of ordinary or partial differential equations (ODEs/PDEs), whose dynamic behaviour – the **conflict dynamics** – and stability properties are analysed by solving the systems analytically or by phase diagramme techniques. Other examples for such models include the Lotka-Volterra-Goodwin equations of predator-prey conflict (Goodwin, 1967), the Intriligator-Brito (1986) model and the Richardson (1919) equations, which were extended in monographs published poshumously (Richardson 1960a, 1960b) as well as in Boulding (1962).[1] The latter model is in the focus of the present analysis, and we will refer to it as the Boulding-Richardson (or B-R for short) model.

One serious lack of this early group of models is that they are effectively "macro" ones (even when used to illustrate the behaviour of individual parties to a conflict) with no microeconomic foundations, i.e. they do not contain explicit optimisation. It comes as no surprise then that the rise of game theory that began in the 1940s led to a shift of interest away from the first generation of models and towards dynamic games and, in particular, differential games (Isaacs, 1954). As far as static patterns of conflict are concerned, game theory provided a convincing taxonomy (Rapoport and Guyer, 1966),[2] and dynamic game theory yielded deep insight into such features of conflict dynamics as the initiative, signals, and reputation.

However, a general taxonomy of conflict dynamics proved elusive, and differential game theory hit a conceptual wall when dealing with the non-linearities that are pervasive in conflict theory (Beckmann and Reimer, 2014). These problems as well as improvements in the raw computing power available to scholars led to increased reliance on simulation methods (Fontana, 2006). And for simulation purposes, both the aforementioned theoretical limits and the insights of behavioural economics recommend some version of boundedly rational optimising. It is in this context that first-generation models may return as more than just a subject for the historian of economic thought.

In the present paper, we propose to re-visit the B-R equations and provide some alternative formulations of the two-party model. We will also discuss the model's

[1]Lewis Fry Richardson is little known in economics, and without the work of his fellow Quaker Boulding his work may not have resounded in our field at all. He is, however, well remembered for his contributions to other disciplines. On this, see Hunt (1995). In mathematics, Richardson's equations are a popular simple model of conflict, which amongst other things is used in the classroom to explain phase diagramming, see `https://www.youtube.com/watch?v=e3FfmXtkppM`. The recent conflict economics text by Anderton and Carter (2009) also has a section on the Richardson model.

[2]See also the recent book by Robinson and Goforth (2005).

application to an empirical example. The ultimate aim of the analysis is to assess whether the B-R model can be used as a building block for modern conflict simulation, and for a taxonomy of dynamic conflict. *En passant*, we may also contribute to the history of economic thought.

The obvious extension to three or more parties, which can also be found in Richardson (1960a), is left to subsequent work. To justify this, we can in the first instance refer to the dominance of the two-party case in much of the economics of conflict.[3] However, we also note that the three-plus-case differs from the two-party one in a very important way, namely that one of the parties can act as an *attenuator*, trying to dampen the conflict between the remaining agents (Goldstein and Pevehouse 1997). All in all, we think that this extension had best be treated seperately.

We begin by re-stating the Richardson equations and illustrating the dynamic properties of the original model (section 2). We find that the standard formulation is deficient in two respects, one having to do with the **psychological** predominance of escalation over the level of aggression, the other dealing with the probabilistic nature of escalation. These extensions and modifications of the BR-model are discussed in turn in section 3, noting that the resulting dynamics are both more stable and more plausible. Section 4 illustrates applicability of the BR model using GDELT data on the Ethiopian-Eritreian war of 1998-2000, and section 5 concludes.

2. The basic B-R model

Richardson (1916) conceived of his equations as a model of an arms race (see also Anderton and Carter 2009, pp. 199-202). In Boulding's (1962) version, the equations describe the joint dynamics of the *aggressiveness* (or escalation level) of two parties to a conflict. This is the story we will adopt here.

Denote by a (b) a measure of party A's (B's) aggressiveness towards the other and assume that without interaction, this reverts over time to a base level $\hat{a}$ ($\hat{b}$). This base level is, however, not the long-term equilibrium because of the interaction effect: each party's aggressiveness increases exponentially as a function of the competitor's escalation measure. Together with the assumption $a, b > 0$, this gives the Richardson equations

$$\dot{a} = k_a(\hat{a} - a) + r_a b \tag{1}$$

[3]Two-party interactions are prevalent among the models presented in Hirshleifer (2001) as well as in the volumes edited by Sandler and Hartley (1995, 2007) or Garfinkle and Skaperdas (2012).

$$\dot{b} = k_b(\hat{b} - b) + r_b a \tag{2}$$

where the strictly positive parameters k_i and r_i represent the parties' speed of adjustment to the base level and sensitivity to aggression, respectively.

We can explicitly solve this system of linear ODEs for the time paths $a(t), b(t)$ of aggressiveness. For example, in the symmetric case where $r_a = r_b = r$ and $k_a = k_b = k$ (assuming $r \neq k$), we find

$$a(t) = \frac{k(\hat{a}k + \hat{b}r)}{k^2 - r^2} + \mathrm{e}^{-kt}(c_1 \cosh(tr) + c_2 \sinh(tr)) \tag{3}$$

where c_1 and c_2 are constants. If we additionally assume that $a(0) = b(0) = 0$, we have

$$a(t) = \frac{e^{2rt} - e^{(k+r)t}}{\frac{r-k}{k} e^{(k+r)t}} \hat{a} \tag{4}$$

and likewise for b.

However, the general properties of this model are better studied using phase diagramme techniques. Letting $\dot{a} = 0$ and $\dot{b} = 0$, we obtain the first two isoclines where the vector field is vertical and horizontal, respectively (written as functions of a for easier plotting)

$$b = \frac{k_a}{r_a}(a - \hat{a}) \tag{5}$$

$$b = \hat{b} + \frac{r_b}{k_b} a \tag{6}$$

Note that both graphs are upward sloping lines in (a, b)-space and that the equation for $\dot{a} = 0$ has a negative intercept on the b axis, while the other cuts the ordinate at $\hat{b} > 0$. This already implies that there are just two possible configurations (see figure 1). If $\frac{k_a}{r_a} > \frac{r_b}{k_b}$, the two lines intersect in the positive orthant (left-hand panel in figure 1) and there exists a stable stationary equilibrium at

$$(a^*, b^*) = \left(\frac{k_b(\hat{a}k_a + \hat{b}r_b)}{k_a k_b - r_a r_b}, \frac{k_a(\hat{b}k_b + \hat{a}r_b)}{k_a k_b - r_a r_b} \right) \tag{7}$$

Otherwise, there is no such intersection and aggressiveness explodes in the positive orthant (right-hand panel in figure 1). Onserve that a *symmetry* assumption, i.e. $r_a = r_b$ and $k_a = k_b$, would generate a borderline case where the graphs of are parallel. The consequences are much the same as in case 2 above, with an explosion of aggression in the first orthant.

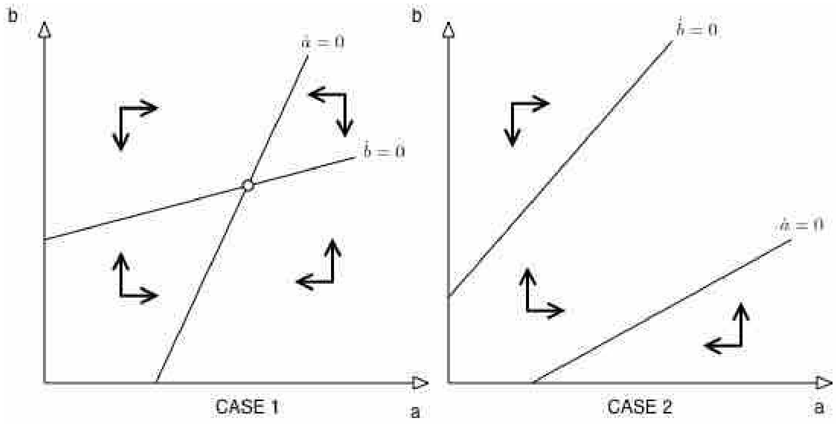

FIGURE 1. The two possible scenarios in the B-R model

Formally, note that the Jacobian for the system (1) and (2) is

$$J = \begin{pmatrix} -k_a & r_a \\ r_b & -k_b \end{pmatrix}$$

with the two eigenvalues $\lambda_{1,2} = -\frac{1}{2}(k_a + k_b \pm \sqrt{(k_a - k_b)^2 + 4r_a r_b})$. As the term under the square root must be positive given our assumptions, both eigenvalues are real. The obvious condition for both eigenvalues to be *negative* is

$$k_a + k_b > \sqrt{(k_a - k_b)^2 + 4r_a r_b}$$

Square both sides of this inequality and rearrange to find $k_a k_b > r_a r_b$, which is equivalent to the graphical restriction on slopes given earlier as a condition for stability. If this inequality does not hold, we will have two real eigenvalues with differing signs, i.e. saddlepoint stability (however, the equilibrium will be in the negative orthant).

The endless escalation of conflict in this case (2) may appear implausible because infinite aggression levels are an unwieldy concept. However, in interpreting the B-R model, one can assume that there exists a threshold level of escalation beyond which the conflict in question changes its nature (i.e., an open outbreak of military hostilities). One can also add an additional constraint to the model – for instance,

a and b could represent the *share* of two competing news media (total broadcast time or pages in a magazine) devoted to a particular conflict, or a particular scandal. The latter modification would give rise to a stable corner solution.

3. Extensions and variations

We now propose two variants of the B-R model, which we explore in turn:

1. a version which incorporates the idea that it may be escalation rather than the stock of aggressiveness which determines the interaction effect,
2. a model which replaces the deterministic interaction effect with a probabilistic version, taking account of Clausewitzian friction and other sources of uncertainty.

3.1. **An incrementalist B-R model.** In our first variation on the B-R theme, we recognise that it can be the *change* in enemy aggression levels, i.e. the *escalation* of conflict, which drives conflict dynamics. We retain the assumption that aggression levels will return to base values $\hat{a}, \hat{b}$ over time, but replace the stock levels of aggression with their time derivatives $\dot{a}, \dot{b}$. This leads to the following model:

$$\dot{a} = k_a(\hat{a} - a) + r_a\dot{b} \tag{8}$$

$$\dot{b} = k_b(\hat{b} - b) + r_b\dot{a} \tag{9}$$

As was the case for the baseline model, we can solve this system of differential equations explicitly, obtaining complete time paths for the two variables of interest, given the parameters and starting values $a(0), b(0)$. Using the symmetric example from section 2, we find

$$a(t) = \left(1 - e^{\frac{k(1+r)t)}{r^2-1}}\right) a(0) \tag{10}$$

with an analogous solution for b. Again, however, we find it more instructive to take a conventional approach using phase diagrammes to illustrate system behaviour over time for more general parameter values.

Substituting $\dot{b}$ into the first equation of the model and rearranging, we can express the change in a and in b as a function of the state variables

$$\dot{a} = \frac{k_a(\hat{a} - a) + r_a k_b(\hat{b} - b)}{1 - r_a r_b} \tag{11}$$

$$\dot{b} = \frac{k_b(\hat{b} - b) + r_b k_a(\hat{a} - a)}{1 - r_a r_b} \tag{12}$$

Proceeding as before, we obtain the following equations for the loci of $\dot{a} = 0$ and $\dot{b} = 0$, respectively.

$$b = \frac{\hat{a}k_a + \hat{b}k_b r_a - k_a a}{k_b r_a} \tag{13}$$

$$b = \frac{\hat{b}k_b + \hat{a}k_a r_b - k_a r_b a}{k_b r_a} \tag{14}$$

Solving this simple system yields the stationary point at $a^* = \hat{a} \wedge b^* = \hat{b}$. This implies that contrary to the standard B-R model, the stationary point always lies in the positive orthant.

For a graphical analysis, observe that the slope of the graph for $\dot{a} = 0$ is steeper than the other iff $r_b < 1$. Also note that the denominator in both equations of motion (11) and (12) becomes *negative* for $r_a r_b > 1$. All in all, this leaves us with four possible dynamic configurations shown in figure 2 below. Case 1 exhibits a stable stationary state, whereas case 2 is characterised by instability. However, case 2 differs from the unstable case in the original model in that a corner solution at the origin is also a possibility. Cases 3 and 4 – where $r_b > 1$ – have saddlepoint stable equilibria.

Start from equations (11) and (12) to find the Jacobian

$$J = \begin{pmatrix} -\frac{k_a}{1-r_a r_b} & -\frac{r_a k_b}{1-r_a r_b} \\ -\frac{r_b k_a}{1-r_a r_b} & -\frac{k_b}{1-r_a r_b} \end{pmatrix}$$

and the two eigenvalues $\lambda_{1,2} = \frac{k_a + k_b \pm \sqrt{(k_a - k_b)^2 + 4k_a k_b r_a r_b}}{2r_a r_b - 2}$. While we can rule out complex eigenvalues again, the fact that the sign of the denominator reverses at $r_a r_b = 1$ now gives rise to a total of four possible configurations, as shown in figure 2 above.

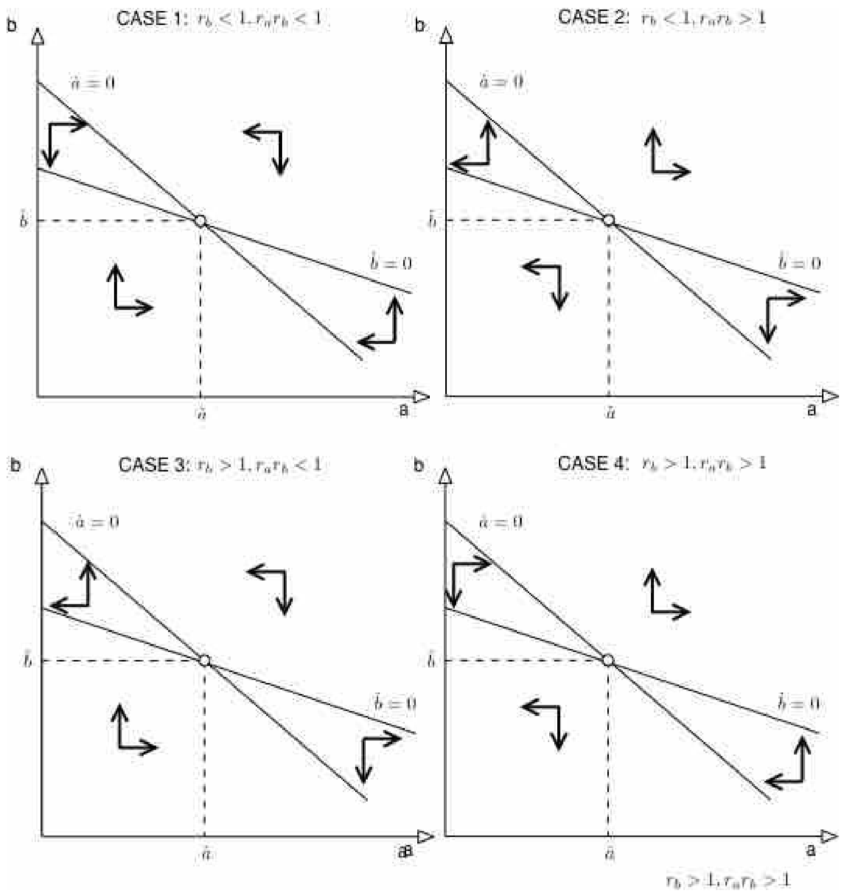

FIGURE 2. The four scenarios in the incrementalist BR model

In the original B-R model, it was the *relative* size of adaptation k and reaction coefficients r that determined the dynamic pattern of conflict. Now, it is the *absolute* value of the reaction coefficients alone that proves crucial. It is sufficient for convergence to a stable equilibrium at the "normal" aggro level $\hat{a}, \hat{b}$ that both parties do not respond "in kind" to an enemy escalation, but with an $r < 1$. This feature of the model appears more plausible than the results we obtained for the original formulation. In addition, the incrementalist model allows for a "pacifist" party (with low r) to compensate for the existence of an aggressive opponent in a very plausible manner.

3.2. **Probabilistic interaction.** Finally, let us briefly consider how to incorporate *randomness* – and Clausewitzian "friction" – into the simple framework. As a large conflict unfolds, there will be several small interactions during which either side can either escalate, de-escalate, or ignore the other side's aggression. Let a's probability p of escalation depend on b's aggro level according to a probability function $p(b)$ with $p' > 0$ and vice versa. For a large number of such interactions per unit of time, the BR equations of motion can then be amended by just plugging in the probability functions for $r_a b$ and $r_b a$, respectively. We then obtain the following system of equations

$$\dot{a} = k_a(\hat{a} - a) + s_a p(b) - s_a(1 - p(b)) \tag{15}$$

$$\dot{b} = k_b(\hat{b} - b) + s_b p(a) - s_b(1 - p(a)) \tag{16}$$

where the s_i represent party i's "step size" of (de-)escalation, assumed to be a constant for simplicity.

We require a specific probability function for plotting or explicit solutions, although basic phase diagrammes such as the ones in figures 1 and 2 could by derived with just some assumptions regarding the curvature of p. Borrowing from the literature on conflict success functions,[4] we employ a *logistic function*

$$p(a) = \frac{1}{1 + e^{\kappa(\hat{a} - a)}} \tag{17}$$

where $\hat{a}$ denotes the reference level of aggression by A (i.e., the level where escalation and de-escalation are just as likely), and the parameter κ determines the steepness of the probability function.

One important difference from the variants discussed previously is that the isoclines for $\dot{a}, \dot{b} = 0$ are now non-linear. Also, the fact that the limits of the logistic function are zero for $a, b \to -\infty$ and one for $a, b \to +\infty$ together with the structure of the system imply that there exists a stable intersection in the positive orthant. Figure 3 below illustrates this for the symmetric case.[5]

4. Empirical illustration

In order to provide an empirical illustration for the suitability (or otherwise) of the B-R model, two prerequisites need to be met: first, one has to find a well-documented conflict in history with just two parties to it, and second, the state variables of the model need to be identified in the appertaining dataset.[6]

[4]The classic treatment is the book by Hirshleifer (2001).

[5]We assume a symmetric solution with the following parameter values: $k_a = k_b = \frac{1}{5}, r_a = r_b = \frac{1}{2}, \hat{a} = \hat{b} = 10$. The plot was produced using *Mathematica*.

[6]We recognise in passing that Richardson (1960b) also played a pioneering role in the systematic collection of data about conflicts.

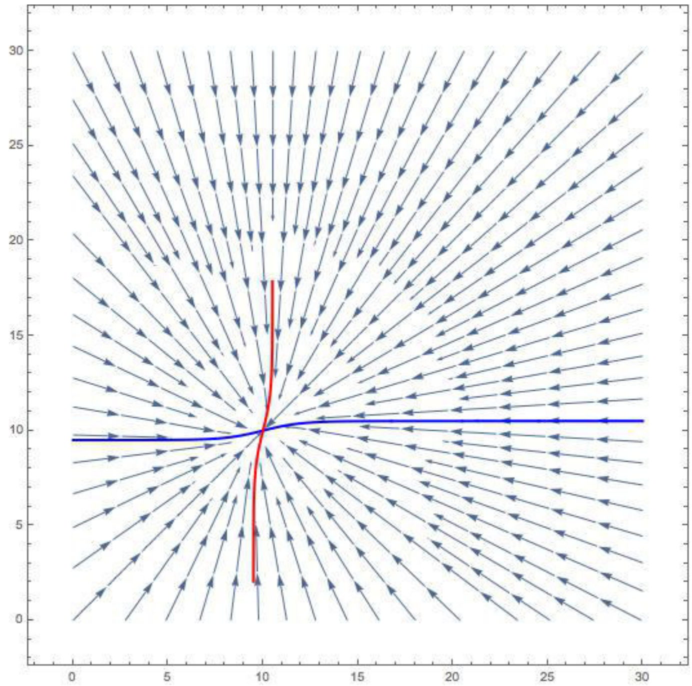

Figure 3. A stable stationary point in the probabilistic model

Regarding the first issue, we focus on the war between Ethiopia and Eritreia (hostilities lasted from May 1998 to May 2000,[7] but our dataset includes the three years preceding the outbreak of hostilities and following the ceasefire), arguing that this is indeed a conflict in which external players and mediators did not play a decisive role.

As to the second, we follow a large strand of the literature using the GDELT[8] database of coded discrete event data (see Goldstein and Pevehouse, 1997, 1999). Our measure of *(de-)escalation* or the *change of aggressiveness* is the Goldstein score (Goldstein 1992), which assigns each conflictary (and cooperative) action an integer in the interval $[-10; 10]$ indicating the flow impact on relations between the involved parties. The state variables, therefore, are just the sum of the (undiscounted) Goldstein scores accumulated over the course of the conflict. One unfortunate consequence of this is that we need to fix the starting values for the state variables $a(0), b(0)$ at some arbitrary level – zero in the following illustration.

[7]See `https://en.wikipedia.org/wiki/EritreanEthiopian_War`.

[8]The GDELT Project – Global Database of Events, Language and Tone – `http://gdeltproject.org/`.

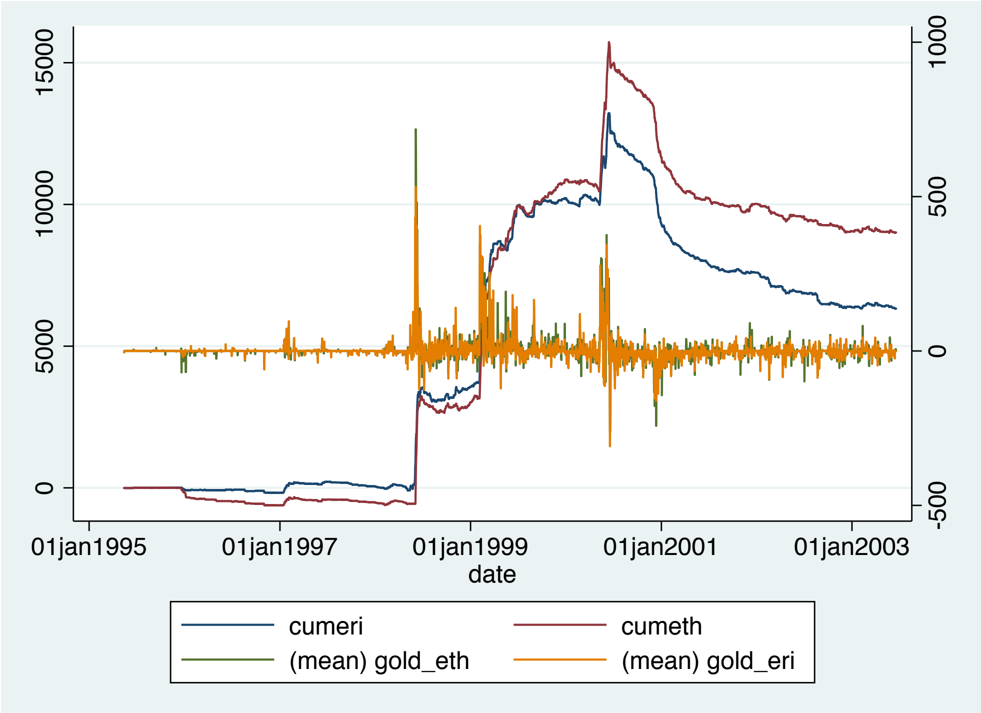

FIGURE 4. Goldstein (1992) scores and "Goldstein levels" for the Ethiopian-Eritreian conflict, 1995-2002

This being said, figure 4 above plots the four time series – our measures for $\dot{a}, \dot{b}, a, b$ – over time.[9]

On inspection of figure 4, we observe that the accumulated Goldstein scores remained constant in the years preceding the war and abated in the aftermath of the ceasefire, we also observe a (not unexpected) steep increase during hostilities. The change of aggressiveness during the war is dominated by three extreme peaks corresponding to major campaigns[10] and also influenced by the onset of the rain season, which impeded the movement of motorised troops. It is clear that such peaks of escalation are incompatible with the B-R model. This bolsters Richardson's original arms race story relative to Boulding's psychological version, which might be applied to wartime aggression as well. It is also compatible with the hypothesis that open hostilities arising whenever the stock of aggression exceeds an exogenous threshold level in an unstable B-R model.

[9]The two flow variables are called "golderi" – the sum of the Goldstein indices assigned to Eritreia's actions towards Ethiopia on a given day – and "goldeth", while we refer to the stock variables as "cumeri" and "cumeth".

[10]The Eritrean attack on Badme in May 1998 including the subsequent air war, Ethiopia's offensive of February 1999, and the final Ethiopian attack in May 2000 that severed Eritreian lines of communication and paved the way for the ceasefire.

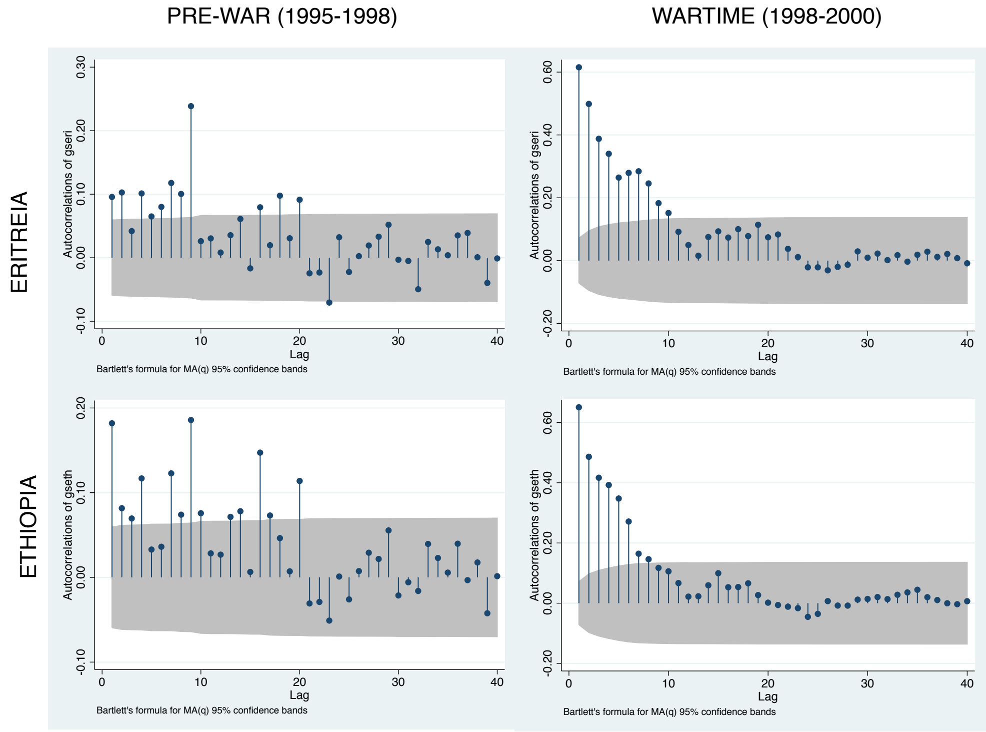

Figure 5. Correlograms for our time series of Goldstein scores

Evidence of an additional difference between the shooting war and the period of time preceding it can be found by looking at the correlograms of the Goldstein score time series separately for the war and the three years leading up to it (figure 5). Not only does there seem to be more autocorrelation during the war, but significant lags appear clustered over the first week. This is suggestive of the effect of military planning leading to continuous activity. During the crisis before the war, on the other hand, significant lags were not concentrated in the same manner, and the correlation coefficients do not shrink over time as they do during the war.

Having observed differences between the wartime and pre-/post-war times series and recognised that the B-R model appears comparatively less attractive as a framework for modelling the former, we now proceed to a formal time series analysis of our data set. Table 1 on page 12 summarises the results. An augmented Dickey-Fuller test using 21 lags as suggested by the Schwert criterion allows us to reject the null of a unit root for our time series of Goldstein scores (analysing the pre-war, wartime and post-war periods separately). Unsurprisingly, this is not the case for the state variable, i.e. the accumulated scores (see table 1). We use a Johansen test for cointegration – again with 21 lags – and find that the time series for the Goldstein scores are clearly cointegrated at all conventional levels

TABLE 1. Summary of time series analyses

Type of analysis	Time series	1995-1998	1998-2000	2000-2003
Stationarity	goldeth	yes***	yes***	yes***
	golderi	yes***	yes***	yes***
	cumeth	no	no	no
	cumeri	no	no	no
Cointegration	golderi, goldeth	yes***	yes***	yes***
	cumeri, cumeth	no	no	yes***
Granger causality	golderi ⇒ goldeth	yes***	yes***	yes***
	goldeth ⇒ golderi	yes***	yes***	yes***
	cumeth ⇒ golderi	yes***	yes***	yes***
	cumeri ⇒ goldeth	yes***	yes***	yes***
	cumeth ⇒ goldeth	no	no	no
	cumeri ⇒ golderi	no	no	no

of significance, while no significant evidence of cointegration can be found for the accumulated scores with the interesting exception of the post-war period.

The most interesting (non-)results can be found in the last two lines of table 1: a conflict party's accumulated Goldstein scores *do not Granger cause their daily escalation.* Regardless of which variant of B-R model from sections 2 and 3 one chooses, this effect is a clear implication. We thus conclude that the data on the conflict at hand are not consistent with the B-R approach.

5. Conclusion

The present paper revisits the B-R model from both an analytical and an empirical perspective. In so doing, our objective is to see whether this old staple can be brought back from the world of teaching (where it serves as an example for solving systems of differential equations) into modern research on conflict dynamics.

In the analytical part, we find that the dynamic properties of the model can be improved upon by letting a party's level of aggression depend on the *change* in the other party's aggro level ("escalation") rather than on the stock variable. We also note that the probabilistic nature of conflict, which has figured prominently in the military literature since Clausewitz' (1873) made friction a core element of his theory, and which is also reflected by the modern vision of "hybrid" warfare, can be integrated into the BR model. Such integration can be shown to improve the stability properties of the BR model.

The negative findings in the empirical section could arise in at least two ways: first, the Goldstein scores from databases like GDELT may be an inappropriate measure of aggressiveness levels and escalation, and second, the B-R model may not fit the situation at hand. Given the successful empirical work using Goldstein scores (e.g., Goldstein and Pevehouse 1999), we tend to favour the latter explanation.

Recall that Richardson (1919, 1960a) explicitly developed his equation to model *arms races*. It was Boulding (1962) who added a psychological interpretation, modelling how one party's aggression level depends on the *perceived* aggression by the other party, and vice versa. Our empirical results cast doubt on this particular application of the B-R equations. In particular, we raised two objections:

(1) The model does not fit the pattern of escalation and de-escalation in a shooting war. One problem is that the grouping of actions into (military) operations and the concomitant constraints – in our example: the Monsoon – is not captured.

(2) Any "regression to the mean" of the state variable would imply a causal link between this state and later (de-)escalation, which we fail to find in our time series of daily sums of Goldstein scores.

This criticism need not impinge on using the B-R model as it was originally designed, or for applications that closely resemble arms races. The treatment of scandals by the media may serve as an example.

Bibliography

Anderton, Charles H., and John R. Carter. 2009. Principles of Conflict Economics. Cambridge: Cambridge UP.

Beckmann, Klaus B., and Lennart Reimer. 2014. Dynamics of Military Conflict: an Economics Perspective. Review of Economics 65, 265-285.

Boulding, Kenneth E. 1962. Conflict and Defence: A General Theory. Harper & Bros.

Clausewitz, Carl von. 1873. On War. London: N. Trübner.

Fontana, Magda. 2006. Computer simulations, mathematics and economics. RISEC 53, no. 1: 96-124.

Garfinkel, Michelle R., and Georgios Skaperdas (eds.). 2012. The Oxford Handbook of the Economics of Peace and Conflict. Oxford: Oxford UP.

Goldstein, Joshua S. 1992. A Conflict-Cooperation Scale for WEIS Events Data. The Journal of Conflict Resolution, Vol. 36, No. 2 (Jun., 1992), pp. 369-385.

Goldstein, Joshua S., and J.C. Pevehouse. Reciprocity, Bullying and International Cooperation: Time-series Analysis of the Bosnia Conflict. The American Political Science Review, Vol. 91, No. 3 (Sep., 1997), pp. 515-529.

Goldstein, Joshua S., and J.C. Pevehouse. Serbian Compliance or Defiance in Kosovo? Statistical Analysis and Real-Time Predictions. The Journal of Conflict Resolution, Vol. 43, No. 4 (Aug., 1999), pp. 538-546.

Goodwin, R.M. 1967. A growth cycle. In Socialism, Capitalism and Economics Growth, ed. C.H. Feinstein, Cambridge University Press.

Hirshleifer, Jack. 2001. The Dark Side of the Force. Economic Foundations of Conflict Theory. Cambridge: Cambridge University Press.

Hunt, J.C.R. 1995. Lewis Fry Richardson and his contributions to mathematics, meteorology, and models of conflict. Annual Review of Fluid Mechanics 30, xiii-xxxvi.

Intriligator, Michael D., and Dagobert L. Brito. 1986. Arms races and instability. Journal of Strategic Studies 9(4), pp. 339-353.

Isaacs, Rufus. 1954. Differential Games: A Mathematical Theory with Applications to Warfare and Pursuit, Control and Optimization.

Lanchester, Frederick W. 1956. Mathematics in warfare. In The World of Mathematics, ed. J.R. Newman, 2138-2157. New York: Simon and Schuster.

Rapoport, Anatol, and M. Guyer. 1966. A taxonomy of 2 x 2 games. General Systems 11, 203-214.

Richardson, Lewis Fry. 1919. The Mathematical Psychology of War. Oxford: Hunt.

Richardson, Lewis Fry. 1960a. Arms and Insecurity. (Edited by Nicolas Rashevsky and Ernesto Trucco.) Chicago: Quadrangle Books.

Richardson, Lewis Fry. 1960b. Statistics of Deadly Quarrels. (Edited by Quincy Wright and C.C. Lienau.) Chicago: Quadrangle Books.

Robinson, David, and David Goforth. 2005. The Topology of 2x2 Games: A New Periodic Table. Routledge Advances in Game Theory. London: Routledge.

Sandler, Todd, and Keith Hartley (eds.). 1995. Handbook of Defense Economics. Vol. 1. Amsterdam et al.: Elsevier.

Sandler, Todd, and Keith Hartley (eds.). 2007. Handbook of Defense Economics. Vol. 2. Amsterdam et al.: Elsevier.

Helmut-Schmidt-Universität (UniBw H), Holstenhofweg 85, 22043 Germany.

E-mail address: klaus.beckmann@hsu-hh.de

URL: http://www.hsu-hh.de/beckmann/

Helmut-Schmidt-Universität (UniBw H), Holstenhofweg 85, 22043 Germany.

E-mail address: gattke@hsu-hh.de

Helmut-Schmidt-Universität (UniBw H), Holstenhofweg 85, 22043 Germany.

E-mail address: reimerl@hsu-hh.de

2015

158 Jahn, Vera: The Importance of Mittelstand Firms for Regional Apprenticeship Activity - Lessons for Policy, April 2015

157 Im Winkel, Niklas: Rechts? Links? Liberal? Egal? Gründe für die Entstehung verzerrter Medieninhalte und Methoden zur Messung des Bias, February 2015

156 Afflatet, Nicolas: Public Debt and Borrowing. Are Governments Disciplined by Financial Markets?, January 2015

2014

155 Berlemann, Michael; Christmann, Robin: Determintants of In-Court Settlements. Empirical Evidence from a German Trial Court, December 2014

154 Berlemann, Michael; Christmann, Robin: Do Judges React to the Probabilty of Appellate Review? Empirical Evidence from Trial Court Procedures, December 2014

153 Bennöhr, Lars; Oestmann, Marco: Determinants of house price dynamics. What can we learn from search engine data?, October 2014

152 Dewenter, Ralf; Giessing, Leonie: The Effects of Elite Sports on Later Job Success, October 2014

151 Dewenter, Ralf; Rösch, Jürgen; Terschüren, Anna: Abgrenzung zweiseitiger Märkte am Beispiel von Internetsuchmaschinen, October 2014

150 Berlemann, Michael; Jahn, Vera: Governance, firm size and innovative capacity: regional empirical evidence for Germany, August 2014

149 Dewenter, Ralf; Rösch, Jürgen: Net neutrality and the incentives (not) to exclude competitors, July 2014

148 Kundt, Thorben: Applying “Benford’s“ law to the Crosswise Model: Findings from an online survey on tax evasion, July 2014

147 Beckmann, Klaus; Reimer, Lennart: Dynamiken in asymmetrischen Konflikten: eine Simulationsstudie, July 2014

146 Herzer, Dierk: Unions and income inequality: a heterogeneous panel cointegration and causality analysis, July 2014

145 Beckmann, Klaus; Franz, Nele; Schneider, Andrea: Intensive Labour Supply: a Menu Choice Revealed Preference Approach for German Females and Males, June 2014

144 Beckmann, Klaus; Franz, Nele; Schneider, Andrea: On optimal tax differences between heterogenous groups, May 2014

143 Berlemann, Michael; Enkelmann, Sören: Institutions, experiences and inflation aversion, May 2014

142 Beckmann, Klaus; Gattke, Susan: Tax evasion and cognitive dissonance, April 2014

141 Herzer, Dierk; Nunnenkamp, Peter: Income inequality and health – evidence from developed and developing countries, April 2014

140 Dewenter, Ralf; Heimeshoff, Ulrich: Do Expert Reviews Really Drive Demand? Evidence from a German Car Magazine, March 2014

139 Dewenter, Ralf; Heimeshoff, Ulrich: Media Bias and Advertising: Evidence from a German Car Magazine, March 2014

138 Beckmann, Klaus; Reimer, Lennart: Dynamics of military conflict from an economics perspective, February 2014

2013

137 Christmann, Robin: Tipping the Scales - Conciliation, Appeal and the Relevance of Judicial Ambition, December 2013

136 Hessler, Markus; Loebert, Ina: Zu Risiken und Nebenwirkungen des Erneuerbare-Energien-Gesetzes, June 2013