Reinvestigating the uniqueness of price equilibria in a product positioning model under price competition in a multisegmented market†

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Abstract

We reinvestigate a theoretical result by Rhim & Cooper (Int. J. Res. Mark. 22(2), 2005, 159–182), who provide a uniqueness condition for price equilibria in a two-stage competitive product positioning model. We show that this condition is very restrictive by providing a simple proof for the fact that it eventually results in parametric pricing. As a consequence, it misses the two-stage characteristic that the model originally aims at.

Keywords: Product positioning, Price competition, Nash equilibrium

1. Introduction

When incorporating simultaneous price competition into product positioning models, one typically needs to determine Nash equilibrium pricing strategies (see, for example, [1, 5]). This, however, may well be a daunting task, as the structure of the problem may prohibit "easy" derivation of closed form (and non-trivial) existence or uniqueness results. In this context we reinvestigate a theoretical result by Rhim & Cooper [5], who present a still actively cited model of competitive positioning and pricing of new products of profit maximizing firms in a multisegmented market. As the choice of product positions is typically less flexible than the corresponding pricing decisions, the model is composed of two (interdependent) stages as presented in Figure 1. The first stage is solely concerned with the product positioning decisions of the firms. However, when making their positioning decisions, the firms anticipate that later price competition will affect the firms' profits. This phase of competition in prices corresponds to the second stage of the model: the number of products and their positions are given; simultaneous pricing decisions are made. Hence, this second stage, on which we will focus for the remainder of the paper, is a noncooperative game in which the strategies are prices and the payoffs are profits. A solution to the second stage is a pure strategy Nash

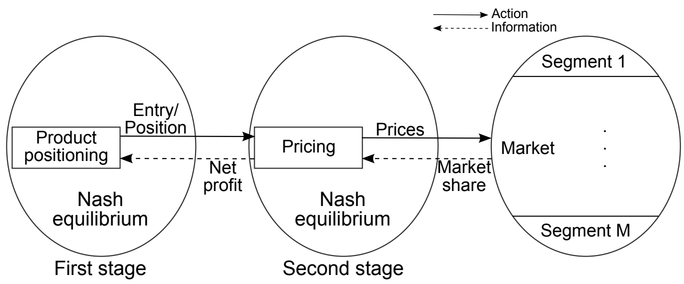

Figure 1: Competitive positioning and pricing framework [5]

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equilibrium in prices, assuming that such an equilibrium exists. In [5], the authors derive a sufficient condition for the existence of a unique Nash equilibrium in prices and state that their "developments throughout [the] paper assume the sufficient condition [to be satisfied]". However, the authors oversee that this condition is very restrictive. If we were to guarantee uniqueness by restricting the parameters of the model to fit this condition, we would enforce a trivial outcome of the pricing game so that prices would eventually become parameters of the model. This is clearly in conflict with the two-stage characteristic under investigation. Hence, if uniqueness needs to be guaranteed, a more general uniqueness condition needs to be derived, which is a potential direction for future research. In what follows, we will provide proof for this claim.

2. Some details on the pricing stage

Rhim & Cooper's setting in the second stage is as follows. N firms have each launched a single product with multiple attributes. The dimension of the product-attribute space is Na. The consumers are grouped into M market segments. The nonnegative demand of market segment $j \in \{1, \ldots, M\}$ is denoted by D_j. The position of product $i \in \{1, \ldots, N\}$ within the product-attribute space is known and denoted by $x(i) = (x(i)_1, \ldots, x(i)_{Na})$. The position of product $i \in \{1, \ldots, N\}$ determines its variable cost, denoted by $c_{x(i)}$, as well as its fixed cost, denoted by $f_{x(i)}$. The price sensitivity of the consumers towards product $j \in \{1, \ldots, N\}$ is represented by the parameter $\gamma_j > 0$. Furthermore, the (nonnegative) distance between product $i \in \{1, \ldots, N\}$ and the ideal point of market segment $j \in \{1, \ldots, M\}$ is denoted by $d_{x(i)j}$. All of these parameters are given when entering the second stage of Rhim & Cooper's model. We are faced with the problem of determining a Nash equilibrium in prices, where the price of product $i \in \{1, \ldots, N\}$ is denoted by $p_{x(i)}$.

The profit π_i of firm $i \in \{1, \ldots, N\}$ is

$$\pi_i = (p_{x(i)} - c_{x(i)}) \sum_{j=1}^{M} D_j MS_{x(i)j} - f_{x(i)}, \tag{1}$$

where $MS_{x(i)j}$ denotes the market share of product $i \in \{1, \ldots, N\}$ in market segment $j \in \{1, \ldots, M\}$. In [5], the authors utilize the multinomial logit model as a probabilistic consumer choice model (cf., for example, [6]), so that

$$MS_{x(i)j} = \frac{A_{x(i)j}}{\sum_{k=1}^{N} A_{x(k)j} + A_{0j}}, \tag{2}$$

for all $i \in \{1, \ldots, N\}$, $j \in \{1, \ldots, M\}$. Here,

$$A_{x(i)j} = e^{-d_{x(i)j} - \gamma_j p_{x(i)}} \tag{3}$$

for all $i \in \{1, \ldots, N\}$, $j \in \{1, \ldots, M\}$, and the finite values $A_{0j} \geq 0$ correspond to the no-purchase options, which are typically set to one.

The price $p_{x(i)}$ of product $i \in \{1, \ldots, N\}$ is assumed to be bounded below by the variable cost $c_{x(i)}$ and bounded above by the reservation price $rp_{x(i)}$ that the consumers have towards the product. Hence, firms set prices on the closed intervals $[c_{x(i)}, rp_{x(i)}]$ for all $i \in \{1, \ldots, N\}$.

3. Uniqueness of Nash equilibria

In [5], the authors prove the following proposition.

Proposition 1 (Rhim & Cooper [5]). *If* $\gamma_j \leq \frac{1}{rp_{x(i)} - c_{x(i)}}$ *for all* $i \in \{1, \ldots, N\}$ *and all* $j \in \{1, \ldots, M\}$*, then there exists a unique pure strategy Nash equilibrium in prices.*

The proof of Proposition 1 is rather lengthy as it is based on a fairly general statement (see [2]). We will now prove an even stronger statement which implies Proposition 1 (see also [4] for a similar model in the facility location context).

Proposition 2. *If*

$$\gamma_j \leq \frac{1}{rp_{x(i)} - c_{x(i)}} \tag{4}$$

for all $i \in \{1, \ldots, N\}$ *and all* $j \in \{1, \ldots, M\}$*, then there exists a unique pure strategy Nash equilibrium in prices with all firms* $i \in \{1, \ldots, N\}$ *charging* $p_{x(i)} = rp_{x(i)}$*.*

Proof. We will show that π_i is strictly monotonic increasing in $p_{x(i)}$ on the interval $I = [c_{x(i)}, rp_{x(i)}]$ if (4) holds. This will prove the claim.

π_i is strictly monotonic increasing on I if

$$\frac{\partial \pi_i}{\partial p_{x(i)}} = \sum_{j=1}^{M} D_j MS_{x(i)j} + (p_{x(i)} - c_{x(i)}) \sum_{j=1}^{M} D_j \cdot \frac{\partial MS_{x(i)j}}{\partial p_{x(i)}} > 0$$

on I. It is easy to see that it is sufficient to require

$$MS_{x(i)j} + (p_{x(i)} - c_{x(i)}) \frac{\partial MS_{x(i)j}}{\partial p_{x(i)}} > 0 \tag{5}$$

for all $j \in \{1, \ldots, M\}$. As shown in [5], we have

$$\frac{\partial MS_{x(i)j}}{\partial p_{x(i)}} = -\gamma_j MS_{x(i)j} \left(1 - MS_{x(i)j}\right). \tag{6}$$

Hence, (5) becomes

$$MS_{x(i)j} - \gamma_j MS_{x(i)j} (p_{x(i)} - c_{x(i)}) \left(1 - MS_{x(i)j}\right) > 0,$$

or, because $0 < MS_{x(i)j} \leq 1$ for all $j \in \{1, \ldots, M\}$ due to the exponential function in (3),

$$1 - \gamma_j (p_{x(i)} - c_{x(i)}) \left(1 - MS_{x(i)j}\right) > 0$$

for all $j \in \{1, \ldots, M\}$. It is easy to see that this holds true due to assumption (4) and the fact that $0 \leq \left(1 - MS_{x(i)j}\right) < 1$ for all $j \in \{1, \ldots, M\}$ because of the exponential function in (3). This concludes the proof. □

Thus, if (4) is fulfilled for all $i \in \{1, \ldots, N\}$ and $j \in \{1, \ldots, M\}$, we have shown that the unique Nash equilibrium in prices is such that the prices of all firms are set to their upper bounds. As these upper bounds are parameters of the model, the prices of all firms are known when the instance of the model is defined. Hence, the model reduces to a pure (one-stage) product positioning game with given (parametric) prices.

We close this research note by taking a closer look at the "exponential assumption" in (3). Making this assumption has multiple important consequences. First, applying the multinomial logit model results in an endogenous upper bound on prices for the product positioning model under consideration, because $\lim_{p_{x(i)} \to \infty} \pi_i \leq 0$ for all $i \in \{1, \ldots, N\}$ [5]. Second, it ensures that $A_{x(i)j}$ is monotonously decreasing in $d_{x(i)j}$ and in $p_{x(i)}$. Additionally, it results in (6) as well as $0 \leq \left(1 - MS_{x(i)j}\right) < 1$, as for instance exploited towards the end of the proof of

Proposition 2. Hence, if we were to drop the exponential assumption and consider a different definition $A'_{x(i)j}$ instead of (3), for example by applying a Huff-like consumer choice model where $A'_{x(i)j}$ is assumed to be a power function [3], we would have to make sure to take these points into account.

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