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Need-Based Justice and Distribution Procedures

Capitulate for Nothing? Does Baron and Ferejohn's Bargaining Model Fail Because No One Would Give Everything for Nothing?

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Capitulate for nothing?

Does Baron and Ferejohn's Bargaining Model Fail Because No One Would Give Everything for Nothing?

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Abstract: The model of legislative bargaining developed by Baron and Ferejohn is widely accepted, widely used and therefore had a great impact on the field. A detailed look reveals that this model is based on assumptions which, in certain situations, lead to everyone voting in favour of a distribution proposal according to which the proposer gets everything and everyone else nothing without any benefit being derived from it (except for the proposer). This result is not plausible, and the assumptions are missing a justification. In this article, the critical assumptions are identified and an alternative is justified. Finally, it is shown why this does not have a significant impact on the overall model (which may be the reason why the critical assumptions have gone unnoticed so far).

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Introduction

The, as I will call it, Baron-Ferejohn-Model (BF) is given by Baron and Ferejohn in *Bargaining in Legislature* (1989). The basic setting is the following: A group bargains about the distribution of a resource. A member of the group is chosen randomly with equal probability to make a proposal for the distribution. The other group members vote on this proposal (the approval of the proposer is assumed). If the proposal is accepted by a majority (including the proposer) in favour of the proposal, the proposal is accepted and the bargaining ends. If the proposal does not receive a majority, a new session begins in which another proposer is chosen randomly with equal probability (it might be the one that is chosen before) and her or his proposal is voted on. This process is repeated until a majority is reached. Baron and Ferejohn give different variations and refinements of the basic setting which are quite important but not relevant for the topic discussed in this article, wherefore I will not discuss them.

The BF-model is widely accepted, widely used and has a great impact. Agranov and Tergiman (2014) refer to the model as "the most popular formal model used to study multilateral bargaining" (p. 75). Following Miller, Montero and Vanberg (2018), it is the "most influential formal model of legislative bargaining" (p. 60). Both articles contain comprehensive references that support these statements.

Therefore, it is very irritating that immediately in the proof of the model's first proposition a precondition can be found which seems anything but plausible, but rather unrealistic and strategically very unfavourable. This raises the questions whether Proposition 1 is only valid under assumptions that are very weak and what effects this has on the entire model.

The first chapter sets out the proposition in question and the relevant part of the proof. The second chapter explains why an assumption in the proof is anything but plausible, which is unrealistic and strategically very unfavourable. The third chapter attempts to answer the question why Baron and Ferejohn included this critical assumption in the proof. The fourth chapter contains good news, because it is shown that the essential statement of Proposition 1 is also valid if the problematic statement in the proof of Proposition 1 is omitted and plausible, realistic and strategically reasonable assumptions are made instead.

Proposition 1

The model starts with and builds on the following Proposition 1:

> „*Proposition 1. A strategy configuration is a subgame-perfect equilibrium for a two-session, n-member (with n odd) legislature […] equal probabilities of recognition if and only if it hast the following form:*
>
> a. *If recognized in the first session, a member makes a proposal to distribute δ/n to any $(n - 1)/2$ other members and to keep $1 - \delta(n - 1)/2n$ for his or her district. If recognized in the second session, a member proposes to keep all benefits.*
> b. *Each member votes for any first-session proposal in which the members receives at least δ/n and votes for any second-session proposal.*
>
> *The first proposal is thus accepted, and the legislature adjourns in the first session.*" (p. 1187)

In summary (without loss of generality I assume $\delta = 1$ for simplification[1]), the strategy for a proposer to win the first session is therefore: Give half of the other players an nth of the overall amount to be distributed (to ensure a majority) and keep the rest for yourself. And for a respondent, the strategy is: Agree if you are offered at least one nth of the overall amount to be distributed.

The proof of Proposition 1 – as well as the following proofs – uses backward induction. This means in general, that you are looking at the last session (or more generally the session in which the expected values do not change or do not change more than a specified amount). From this session, you stepwise conclude to the session before it. The guiding question is: If for a special session s, it must be assumed that the relevant quantities assume a certain value, what does this mean for session s-1 and finally, what does this mean for the first session? Even if this procedure starts with statements about the last session, statements about the first session are actually the objective (Proposition 1 handles the simplest case where there are only two sessions, and is then further generalized to cases with more (potentially infinite) sessions).

The proof begins as follows: „*To establish necessity, note that since the game ends after the second session, the continuation value $v_i(2, g) = 0$ for all (null) subgames g and for all members*.“ (p. 1183) Then follows the, as I will call it,

> Critical Statement (CS): „*The member recognized at the beginning of session 2 can thus successfully propose to take all the benefits, since each member will vote for it because the member's allocation is at least as great as the member's continuation value.*“ (p. 1183)

This means that the respondents would agree to an allocation of 0 in Session 2, because they can't get more than 0 in the sessions afterwards (which do not exist and – this is important – *because* they do not exist and therefore there is a hard-coded setting for the expected value for this sessions. This point is further elaborated below).

The Problem

The Critical Statement (CS) is at least extremely irritating and hardly comprehensible. Why should a respondent agree to a proposal, according to which he or she and all others except the proposer get nothing, while the proposer gets everything? I wouldn't do so. And this is not a personal preference, this applies because it would be strategically extremely unfavourable to follow Baron and Ferejohn. Baron and Ferejohn seem to assume that the respondents in Session 2 have to capitulate, because their continuation value is 0. But the respondents are not in such a weak bargaining position as Baron and Ferejohn ascribe to them, because the proposer also has a continuation value of 0 in Session 2. Without a majority among the respondents, the proposer also goes away empty-handed. This is an aspect that is completely ignored by Baron and Ferejohn, but must be considered, as can be seen easily:

a) If you as a respondent principally agree to a proposal of 0 in Session 2 (as Baron and Ferejohn assume) and the proposer anticipates this, the respondent will get 0 for sure.

b) But if you as a respondent principally do not agree to a proposal of 0 in Session 2 (which in my opinion is appropriate and realistic) and the proposer anticipates this, there is a chance to

[1] δ is a factor that models the costs/disadvantages of playing further sessions of bargaining. It lies between 0 and 1 (and can be determined individually in principle). The greater the (individual) costs/disadvantages associated with further bargaining sessions, the smaller δ is.

get more than 0 (this is not certain as the proposer does not need the agreement of all respondents to achieve a majority, so that the proposer only makes an offer greater than 0 to as many respondents as she or he needs for a majority).

If a respondent follows b), the result can never be worse than if she or he follows a) and if a respondent follows b), the result can be better than if she or he follows a). Therefore, no respondent will follow a) (as Baron and Ferejohn assume) and all instead follow b). And a proposer who anticipates this will not offer 0 to all respondents (as Baron and Ferejohn assume), because she or he can not achieve a majority by this and would go away empty-handed by doing so. A proposer who anticipates this must offer more than 0 to as many respondents as she or he needs for a majority. Thus, she or he would not get all benefits (as Baron and Ferejohn assume).

In result, this is highly problematic as it raises the question whether Proposition 1 is only valid under assumptions that are not plausible, that are completely unrealistic, that are strategically very unfavourable for the respondents and therefore very weak. Moreover, if even the first proposition of a model is so vulnerable, the whole model threatens to fall.

Why the Critical Statement?

Baron and Ferejohn take the Critical Statement surprisingly for granted, they pass without comment over CS and one is left on one's own in order to make the statement accessible. There can be found statements in Baron and Ferejohn that suggest two lines of reasoning. One line is substantive in nature, the other is a simplification due to the presentation.

The substantive line of argumentation

In substance – as I understand it (Baron and Ferejohn are not very explicit about this) –, Baron and Ferejohn make the approval or rejection of a respondent dependent on whether a respondent expects a better outcome if no majority is reached in the current session. If a respondent expects a better outcome in the case that no majority is reached, she or he rejects the proposal, otherwise she or he approves. What a respondent expects is formalised by the continuation value, which is essentially (as I understand it and as far as it is relevant here (again: Baron and Ferejohn are not very explicit about this)) the expectation value for the following session(s) in a mathematical sense. This value for respondents in Session 2 is given as 0 by Baron and Ferejohn and for all players in Session 1 as 1/n (p. 1183). The first holds (I think this is the thinking of Baron and Ferejohn) because all respondents get 0 if no majority is reached in Session 2. The second holds, because based on the first for all players, it is only relevant for their value in Session 1 for session 2 who is chosen to be the proposer in Session 2. If in Session 2 all (respondents), who expect 0 without majority, agree to a proposal of 0, the proposer in Session 2 gets everything. And if all players are just as likely chosen to be the proposer in Session 2, the result is a value of 1/n, which every individual can expect in Session 1 for Session 2. This is at first glance convincing and seems to be appropriate (you may say by definition) for risk neutral players (which are assumed by Baron and Ferejohn (p. 1183)).

But, as shown above, it is not a good strategy to bargain in this way, including for risk neutral players. It is important to understand the difference: If the outcome for the next session(s) is (are) uncertain (or more precisely: risky), risk neutral respondents should follow the textbook advice and play the strategy of voting for a proposal if their allocation is at least as large as they expect for the further session(s). This case is given in the first session. But in the special case, in which the respondents know for sure that they will get nothing if no majority is reached in the current session, and more importantly, that the proposer will get nothing if no majority is reached in the current session, this is a bad advice. This case is given in the second session. The more fundamental reason for this is that it only makes sense to agree to a proposal that gives the respondents the value they expect in further sessions, if their actual pay-out in further sessions can be worse than the offer. If the pay-out cannot be worse if they do not agree, there is no reason to agree. In cases like these, they should change their perspective from their expectation to the expectation of the proposer. If the proposer does not get anything without a majority (or, more generally, less than with a majority), there is a chance for all respondents to get more if they do not play the strategy to agree if they are offered an amount that they would get without agreement. Instead, they must play a strategy that they agree only if they are offered an amount that is greater than the value they would otherwise get. I have the impression that Baron and Ferejohn have not considered this difference.

The simplification due to the presentation

Baron and Ferejohn must face up to this criticism of substance, but they cannot be accused of a formal error. This is, because a few paragraphs before Proposition 1, they comment on how an indifferent player has to decide: „*A member will be assumed to vote for a bill if indifferent between the distribution in the bill and the continuation value of the subgame beginning in the next session.*“ It is therefore consistent that a respondent in Session 2 – if an offer of 0 is matched by an continuation value of 0 – agrees to the offer (but again, as has been explained above, it is not a good strategy to bargain in this way). The justification for this determination is hidden in an end note: „*If a member votes for a bill only if the member strictly prefers it, the distribution necessary to achieve a majority lies in an open set. Then an equilibrium does not exist, since for any distribution that achieves a majority, there is another distribution that also achieves a majority and is better for the proposer. To avoid these complications and the use of an equilibrium concept such as an epsiplon-equilibrium, a member who is indifferent between a bill and continuing to the next session is assumed to vote for the bill.*“

Apart from the fact that it would be absolutely unsatisfactory if the irritations regarding the Critical Statement and a bad strategy were a result from this hidden end note-simplification, this simplification is not necessary at this point, as will be explained in the following chapter.

Solving the problem

Let us assume that a respondent generally disagrees if an offer of 0 is made to her or him (even if the expected value for his future allocations is 0, although this need not be assumed). The proposer would then have to offer ceil((n – 1)/2)[2] (randomly and with equal probability selected[3]) respondents an amount a > 0[4] each in Session 2 to get a majority for her or his proposal. The respondents accept the proposal, as the alternative would be an allocation of 0. There is therefore a majority in favour of this proposal. The proposer keeps a share of 1 - ceil((n – 1)/2) * a for her or himself. Knowing that Session 2 is played that way, the risk-neutral players in Session 1 orient themselves by their expectation value E for Session 2. With a probability of 1/n (this is the probability to be chosen as the proposer) a player will be the proposer and receive

[2] I generalize the statements in a way that they apply to an odd number of players n as well as to an even number of players n.

[3] Baron and Ferejohn write that this applies to „any (n-1)/2 other members“ (where they assume that the number of players n is odd); I assume that they too mean that these players are chosen randomly and with equal probability by the proposer.

[4] a must furthermore be smaller than the expected value 1/n, otherwise it would always be advantageous for a respondent in Session 1 to reject and wait for Session 2.

1 - ceil((n - 1)/2) * a (this is the share of the resource that does not need to be allocated by the proposer in order to win a majority – consider the further explanations). With a probability of ceil((n - 1)/2) * 1/n (this is the probability that, if a player is not chosen as proposer, she or he will receive an offer from the proposer) the other players get a proposal of a from the proposer. And with a probability of floor((n - 1)/2) * 1/n (this is the probability that, if a player is not chosen as proposer, she or he will not receive an offer from the proposer) the other players get a proposal of 0:

$$E = 1/n * (1 - \mathrm{ceil}((n - 1)/2) * a) + \mathrm{ceil}((n - 1)/2) * 1/n * a + \mathrm{floor}((n - 1)/2) * 1/n * 0.$$

This equation can be summarized to an interesting result:

$$E = 1/n * (1 - \mathrm{ceil}((n - 1)/2) * a + \mathrm{ceil}((n - 1)/2) * a)$$

$$= 1/n.$$

This means that, in Session 1, the expected value of all players for Session 2 is 1/n. (If more sessions were played, this value would also apply to them. This value applies to all sessions except the last one) The proposer of Session 1 must therefore offer 1/n to ceil((n - 1)/2) respondents to secure a majority and can keep 1 - ceil((n - 1)/2) * 1/n for her- or himself. The amount, which must be offered in Session 2 by the proposer to as many players as needed for a majority, cancels out and is irrelevant for a decision in Session 1. This result is not trivial because it arises from an interplay of probabilities and pay-outs from the proposer, the respondents who receive an offer, and the respondents who do not receive an offer.

Baron and Ferejohn thus (somewhat accidentally) specify the correct values for the first session, but not for the second session (respectively the values for the second session are only valid under unrealistic conditions). But, in result, the good news is that the, in my opinion, most important statement of Proposition 1 remains even without the critical statement being required: Bargaining with risk neutral players, the proposer offers 1/n to ceil((n - 1)/2) players in the first session and these players accept the offer. Therefore, this has no further impact on the BF-model and therefore a correction of the critical statement has no effect on the further model.

Conclusion

It was shown that even the first proposition of the Baron and Ferejohn-model is based on assumptions that are anything but plausible, very unrealistic and strategically very unfavourable. Nobody would vote for a distribution proposal according to which the proposer gets everything and everyone else nothing. It was investigated what might have led Baron and Ferejohn to make these assumptions, and these potential reasons were found to be problematic. The plausible, more realistic and strategically reasonable alternative presented here can be summarized as follows: Do not vote for any proposal that does not put you in a better position than had you not voted for it, and do not assume that someone else would do so[5]. Finally, it was shown that the main results of the BF-model hold even under the conditions presented here.

[5] This can be seen as a contribution to the question whether and to what extent the often-used indifference argument holds, which (for a risk neutral player) simplified says: If an offer is equal to your expected value, agree (and assume that everyone else is doing so too). In my perception, this argument is used far too often out of embarrassment to avoid a more detailed analysis.

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