Exercises for **Digital Audio Signal Processing**, 2nd Edition
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Chapter N°1: **Introduction to Signals and Systems**

Contents:

- Nyquist sampling theorem and digital signals
- Spectrum analysis
- Digital systems
- Matlab introduction

1. Fundamentals of Digital Signal Processing

With respect to the sampling theorem, a time continuous sine wave $x(t)$ with $f_0 = 1$ kHz has to be discretized

1. What is the smallest sampling rate we can use for this case?
2. Give $y(n)$ and $y(t)$ when the only process done on $x(n)$ is attenuation of gain $g = 0.5$.
3. Consider now $x(t)$ to be a sum of two sine of frequencies f_0 and $3f_0$. For a sampling rate $f_S = 4f_0$ is the signal band limited?
4. What do we have to do if the signal we want to process is not band limited?
5. What happens if we do not respect the sampling theorem? Write a small M-file for this example.

2. Spectrum Analysis with Matlab

1. When do we go from the Z-Transform $X(z)$ of a sequence $x(n)$ to the Discrete-time Fourier Transform DTFT $X(e^{j\Omega})$?
2. How can we distinguish DTFT $X(e^{j\Omega})$ and the Discrete Fourier Transform DFT $X(k)$?
3. What is the Fast Fourier Transform FFT and when do we go from DFT to FFT?
4. What is frequency resolution and how can this be increased?
5. Why do we need to window our time signal before performing the FFT?

3. Digital Systems

1. Important properties of filters: Consider the difference equation $y(n) = y(n-1) + x^2(n-1)$. How can you define the filter order? Is this filter linear, stationary, causal and stable?

2. From the general difference equation $y(n) = \sum_{k=0}^{N-1} a_k x(n-k) - \sum_{k=1}^{M} b_k y(n-k)$ we derive several simple digital filters as follows:

   (a) Simple (unity) gain filter: $y(n) = K \cdot x(n), \quad \text{with} \quad h(n) = K \cdot \delta(n))$

   (b) Pure delay filter: $y(n) = x(n-1)$

   (c) Two-term difference filter: $y(n) = x(n) - x(n-1)$ (Differentiator)

   (d) Two-term average filter: $y(n) = \frac{x(n)+x(n-1)}{2}$

   (e) Three-term average filter: $y(n) = \frac{x(n)+x(n-1)+x(n-2)}{3}$

   Give in form of a table the coefficients, the order of each filter given above and specify which are recursive or nonrecursive.

3. For each of the following filters, state the order and identify the coefficients. Is it possible to have a zero-order IIR filter?

   (a) $y(n) = x(n-2)$

   (b) $y(n) = x(n) - 2x(n-1) + 2x(n-2) + x(n-3)$

   (c) $y(n) = x(n) + y(n-1)$ (Integrator)

   (d) $y(n) = x(n) + 2x(n-1) - 3y(n-1) - 2y(n-2)$

4. A digital filter is described by $y(n) = x(n) + 2x(n-1) - 3y(n-1) - 2y(n-2)$.

   (a) State whether the filter is recursive or not and give the order.

   (b) Derive the transfer function and draw the signal flow graph.

   (c) Is the system stable?

   (d) Give the output sequence if the input is a unit impulse $x(n) = \delta(n)$.

5. A digital system is characterized by the following impulse response:
   $h(n) = \frac{1}{3}\delta(n-1) + \frac{2}{3}\delta(n-2) + \delta(n-3) + \frac{2}{3}\delta(n-4) + \frac{1}{3}\delta(n-5)$

   (a) Show $h(n)$ graphically.

   (b) Draw the signal flow graph and derive the transfer function $H(z)$.

   (c) State the order and stability of the system.

   (d) Calculate the $H(e^{j\Omega})$ (DTFT of $h(n)$) and give the result in form of the magnitude and phase response. What is the group delay of the system?