Exercises for **Digital Audio Signal Processing**, 2nd Edition
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Chapter N°5: **Equalizers**

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1. Design of Recursive Audio Filters

1. How can we design a low-frequency shelving filter? Which parameters define the filter? Explain the control parameters.
2. How can we derive a high-frequency shelving filter? Which parameters define the filter?
3. What is the difference between first- and second-order shelving filters.
4. How can we design a peak filter? Which parameters define the filter? What is the filter order? Explain the control parameters. Explain the Q-factor.
5. How do we derive the digital transfer function?
6. Derive the digital transfer functions for the first-order shelving filters.

2. Parametric Audio Filters

1. What is the basic idea for parametric filters?
2. What is the difference between the Regalia and the Zölzer filter structures? Count the number of multiplications and additions for both filter structures.
3. Derive a signal flow graph for first- and second-order parametric Zölzer filters with a direct-form implementation of the allpass filters?
4. Is there a complete decoupling of all control parameters for boost and cut? Which parameters are decoupled?

3. Shelving Filter: Direct Form

Derive a first-order low shelving filter from a purely band-limiting first-order low-pass filter. Use a bilinear transform and give the transfer function of the low shelving filter.

1. Write down what you know about the filter coefficients and calculate the poles/zeros as functions of V_0 and T. What gain factor do you have if $z = \pm 1$?
2. What is the difference between purely band-limiting filters and the shelving filter?
3. How can you describe the boost and cut effect related to poles/zeros of the filter?
4. How do we get a transfer function for cut case from the boost case?
5. How do we get from low shelving filter to high shelving filter?

4. Shelving Filter: All-pass Form

Implement a first order high shelving filter for boost and cut case with the sampling rate $f_S = 44.1$kHz, the cutoff frequency $f_c = 10$ kHz, and the gain $G = 12$ dB.

1. Define the all-pass parameters and coefficients for the boost and cut cases.
2. Derive from the all-pass decomposition the complete transfer function of the shelving filter.
3. Using Matlab give the magnitude frequency response for boost and cut. Show the result for the case where a boost and cut filter are in a series connection.
4. If the input signal to the system is a unit impulse, give the spectrum of the input and out signal for the boost and cut cases. What result do you expect in this case when boost and cut are again cascaded?

5. Quantization of Filter Coefficients

For the quantization of the filter coefficients different methods have been proposed: direct form, Gold and Rader, Kingsbury and Zölzer.

1. What is the motivation behind this?
2. Plot a pole distribution using the quantized polar representation of a second-order IIR filter

$$H(z) = \frac{N(z)}{1 - 2r\cos(\phi\, z^{-1}) + r^2\, z^{-2}} \tag{1}$$

6. Signal Quantization inside the Audio Filter

Now we combine coefficient and signal quantization.

1. Design a digital high-pass filter (second-order IIR), with a cutoff frequency $f_c = 50$ Hz. (Use the Butterworth, Chebyshev or elliptic design methods implemented in Matlab.)
2. Quantize the signal only when it leaves the accumulator (i.e. before it is saved in any state variable).
3. Now quantize the coefficients (direct form), too.
4. Extend your quantization to every arithmetic operation (i.e. after each addition/ multiplication).

7. Quantization Effects in Recursive Audio Filters

1. Why is the quantization of signals inside a recursive filter of special interest?
2. Derive the noise transfer function of the second-order direct-form filter. Apply a first- and second-order noise shaping to the quantizer inside the direct-form structure and discuss its influence. What is the difference between second-order noise shaping and double-precision arithmetic?
3. Write a Matlab implementation of a second-order filter structure for quantization and noise shaping.

8. Fast Convolution

For an input sequence $x(n)$ of length $N_1 = 500$ and the impuls response $h(n)$ of length $N_2 = 31$, perform the discrete time convolution.

1. Give the discrete time convolution sum formula
2. Using Matlab, define $x(n)$ as a sum of two sinusoids and derive $h(n)$ with Matlab function `remez(..)`.
3. Realize the filter operation with Matlab using:
   - the function `conv(x,h)`
   - the sample-by-sample convolution sum method
   - the FFT method
   - the FFT with overlap-add method
4. Describe FIR filtering with the fast convolution technique. What conditions do the input signal and the impulse responses have to fulfill if convolution is performed by equivalent frequency-domain processing?
5. What happens if input signal and impulse response are as long as the FFT transform length?
6. How can we perform the IFFT by the FFT algorithm?
7. Explain the processing steps
   - for a segmentation of the input signal into blocks and fast convolution;
   - for a stereo signal by the fast convolution technique;
   - for the segmentation of the impulse response.
8. What is the processing delay of the fast convolution technique?
9. Write a Matlab program for fast convolution.
10. How does quantization of the signal influence the roundoff noise behavior of an FIR filter?

9. FIR Filter Design by Frequency Sampling

1. Why is frequency sampling an important design method for audio equalizers? How do we sample magnitude and phase response?
2. What is the linear phase frequency response of a system? What is the effect on an input signal passing through such a system?
3. Explain the derivation of the magnitude and phase response for a linear phase FIR filter.
4. What is the condition for a real-valued impulse response of even length N? What is the group delay?
5. Write a Matlab program for the design of an FIR filter and verify the example in the book.
6. If the desired frequency response is an ideal low-pass filter of length $N_F = 31$ with cutoff frequency $\Omega_c = 0.5\,\pi$, derive the impulse response of this system. What will the result be for $N_F = 32$ and $\Omega_c = \pi$?

10. Multi-complementary Filter Bank

1. What is an octave-spaced frequency splitting and how can we design a filter bank for that task?
2. How can we perform aliasing-free subband processing? How can we achieve narrow transition bands for a filter bank? What is the computational complexity of an octave- spaced filter bank?