Exercises for **Digital Audio Signal Processing**, 2nd Edition
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Chapter N°2: **Quantization**

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1. Quantization

1. Consider a 100 Hz sine wave $x(n)$ sampled with $f_S = 44.1$ kHz, $N = 1024$ number of samples and $w = 3$ bit (word-length). What is the number of quantization levels? What is the quantization step Q when the signal is normalized to $-1 <= x(n) < 1$. Show graphically how quantization is performed. What is the maximum error for this 3 bit quantizer? Write a Matlab code for quantization with rounding and truncation.

2. Derive the mean value, the variance and the peak factor P_F of sequence $e(n)$, if the signal has a uniform probability density function in the range $-\frac{Q}{2} < e(n) < -\frac{Q}{2}$. Derive the signal-to-noise ratio SNR for this case? What will happen if we increase our word-length by one bit?

3. As the input signal level decreases from maximum amplitude to very low amplitudes, the error signal becomes more audible. How can you describe the error calculated above when w decreases to 1 bit? Is the classical quantization model still valid? What can be done to avoid this distortion?

4. Write a Matlab code for a quantizer with $w = 16$ bit with rounding and truncation.

   - Plot the nonlinear transfer characteristic and the error signal when the input signal covers the range $3Q < x(n) < 3Q$.
   - Consider the sinewave $x(n) = A\sin(2\pi\frac{f}{f_S}n), n = 0, \ldots, N-1$ with $A = Q$, $\frac{f}{f_S} = 64/N$ and $N = 1024$. Plot the output signal ($n = 0, \ldots, 99$) of a quantizer with rounding and truncation in the time-domain and the frequency domain.
   - Compute for both quantization types the quantization error and the SNR.

2. Dither

1. What is dither and when do we have to use dither?

2. How do we perform dither and which kind of dither do we have?

3. How do we obtain a triangular high-pass dither and why do we prefer it to other dithers ?

4. Matlab: Generate corresponding dither signals for rectangular, triangular and triangular high-pass.

5. Plot the amplitude distribution and the spectrum of the output $x_Q(n)$ of a quantizer for every dither type.

3. Noise Shaping

1. What is noise shaping and when do we do it?

2. Why is it necessary to dither during noise shaping and how do we do this?

3. Matlab: The first noise shaper used is without dither and assumes that the transfer function in the feedback structure can be first-order $H(z) = z^{-1}$ or second-order $H(z) = -2\,z^{-1} + z^{-2}$. Plot the output $x_Q(n)$ and the error signal $e(n)$ and its spectrum. Show with a plot the shape of the error signal

4. The same noise shaper is now used with a dither signal. Is it really necessary to dither with noise shaping? Where would you add your dither in the flow graph to achieve better results?

5. In the feedback structure we now use a psychoacoustic-based noise shaper which uses the Wannamaker filter coefficients

$$h_3 = [1.623, -0.982, 0.109] \quad (1)$$
$$h_5 = [2.033, -2.165, 1.959, -1.590, 0.6149] \quad (2)$$
$$h_9 = [2.412, -3.370, 3.937, -4.174, 3.353, -2.205, 1.281, -0.569, 0.0847] \quad (3)$$

   Show with a Matlab plot the shape of the error with this filter.